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Blowing in the Wind

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Motivation

- Climate change.

- Sustainable economic growth.
- **Renewable energy sources.**
- Unpredictable, weather-dependent.

Electricity features

- Non-storability (supply and demand must always match).
- Seasonality (higher demand in winter months due to the need of heating and longer use of lights).
- Periodic behaviour (higher demand in the peak time, i.e. Monday to Friday between 8 am and 8 pm).
- Mean reversion (over time the electricity prices will tend to their average).
- Large and heteroscedastic volatility.

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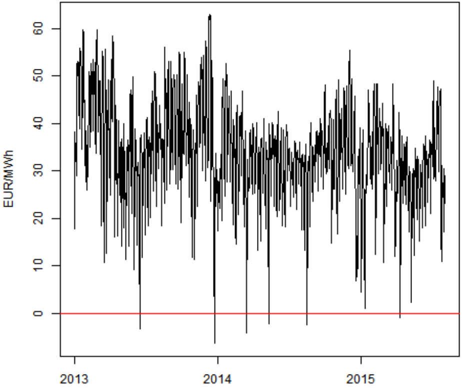
EEX spot prices



Arithmetic model [BKMV14]

$$S(t) = \Lambda(t) + Z(t) + Y(t).$$

- $\Lambda(t) + Z(t)$: long-term factor.
- $Y(t)$: short-term behaviour (includes the impact of renewables).

- $\Lambda(t)$: deterministic seasonality/trend function.
- $Z(t)$: Lévy process with zero mean.
- $Y(t) = \int_{-\infty}^{t} g(t-s)\sigma_{s-} dL_s$ with a deterministic kernel $g(t-s)$ such that $\lim_{t\to\infty} g(t-s) = 0$ (Lévy semistationary process).

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Forward price

By no-arbitrage arguments, one can define the price of a futures contract with a delivery period $[T_1, T_2]$ as

$$F_t(T_1, T_2) := \mathbb{E}_{\mathbb{Q}}\left[\frac{1}{T_2 - T_1}\int_{T_1}^{T_2} S(T)dT \middle| \mathcal{F}_t\right],$$

where $0 \leq t \leq T_1 < T_2$ and $\mathbb{Q}$ is a risk neutral probability measure. The deseasonalised futures price:

$$\tilde{F}_t(T_1, T_2) := F_t(T_1, T_2) - \frac{1}{T_2 - T_1}\int_{T_1}^{T_2} \Lambda(T)dT.$$

Long-term behaviour

- $\lim\limits_{x\to\infty} \int_0^x g(y)e^{-\frac{\delta}{2}(x-y)}dy = 0,\ \delta > 0;$
- $\sigma_t^2 = \int_{-\infty}^t e^{-\delta(t-x)}dV_x;$
- $C := \mathbb{E}_{\mathbb{Q}}\left[\sigma_0\right] \int_0^\infty g(y)dy;$
- $u = \frac{T_1+T_2}{2} - t.$

Then

$$\tilde{F}_t(T_1, T_2) \approx Z(t) + u\mathbb{E}_{\mathbb{Q}}\left[Z(1)\right] + \mathbb{E}_{\mathbb{Q}}\left[L(1)\right] C,$$

where the approximation is in the L^2-sense.

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Empirical studies - the algorithm

1. Estimate $\Lambda(t)$ from the spot prices and subtract from $S(t)$.
2. Filter out a realisation of $Z(t)$.
3. Model $Y(t) = S(t) - \Lambda(t) - Z(t)$ as a CARMA(2,1) process.
4. Add stochastic volatility to the model of $Y(t)$.

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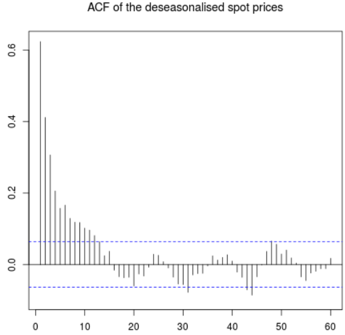
Deterministic trend/seasonality

$$\Lambda(t) = c_1 + c_2 t + c_3 h(t) + \sum_{i=4}^{14} c_i d(t) + \sum_{i=15}^{20} c_i m(t).$$



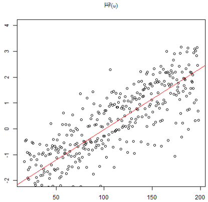
Non-stationary long-term factor $Z(t)$

For big times to maturity u,

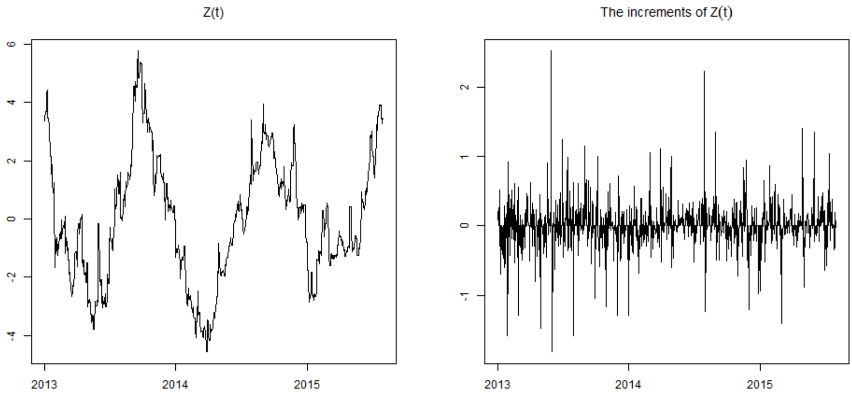
$$\tilde{F}_t(T_1, T_2) \approx Z(t) + u\mathbb{E}_{\mathbb{Q}}[Z_1] + \mathbb{E}_{\mathbb{Q}}[L(1)]\,C.$$

Because $\mathbb{E}[Z(t)] = 0$,

$$\mu_{\tilde{F}}(u) := \mathbb{E}\left[\tilde{F}_t(u)\right] \approx u\mathbb{E}_{\mathbb{Q}}[Z(1)] + \mathbb{E}_{\mathbb{Q}}[L(1)]\,C.$$



A realisation of $Z(t)$



Generalised hyperbolic distributions [Bre11] I

Definition

The random vector X is said to have a multivariate generalised hyperbolic (GH) distribution if

$$\mathbf{X} \overset{law}{=} \boldsymbol{\mu} + W\boldsymbol{\gamma} + \sqrt{W}\mathbf{A}\mathbf{Z},$$

where $\mathbf{Z} \sim N(\mathbf{0}, \mathbf{I}_k)$, $\mathbf{A} \in \mathbb{R}^{d\times k}$, $\boldsymbol{\mu}, \boldsymbol{\gamma} \in \mathbb{R}^d$ and W is a scalar-valued random variable, independent of $\mathbf{Z}$, whose distribution is Generalised Inverse Gaussian.

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Generalised hyperbolic distributions [Bre11] II

Definition

The density of $W \sim GIG(\lambda, \chi, \psi)$ with parameters satisfying one of the following: $\chi > 0, \psi \geq 0, \lambda < 0$ or $\chi > 0, \psi > 0, \lambda = 0$ or $\chi \geq 0, \psi > 0, \lambda > 0$ is given by

$$f_{\mathrm{GIG}}(x) = \left(\frac{\psi}{\chi}\right)^{\frac{\lambda}{2}} \frac{x^{\lambda-1}}{2K_\lambda\left(\sqrt{\chi\psi}\right)} \exp\left(-\frac{1}{2}\left(\frac{\chi}{x} + \psi x\right)\right).$$

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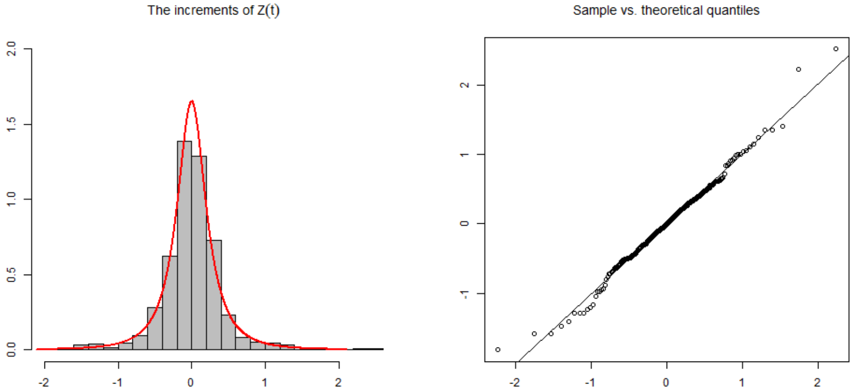
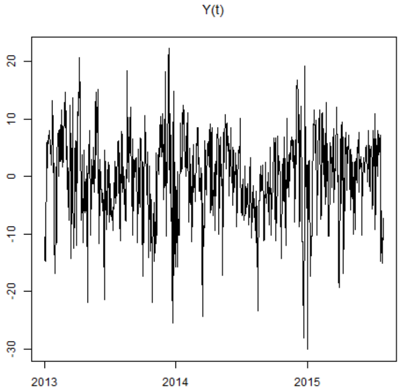
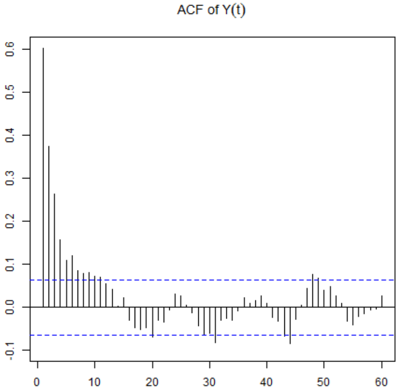
Fitted NIG distribution



Stationary short-term factor $Y(t)$





CARMA [BDY11]

Definition

Assume that $L(t)$ is a second order subordinator and $p > q$. Then we define the L-driven CARMA(p,q) with parameters $[a_1, \ldots, a_p; b_1, \cdots, b_q]$ as the solution of the system of stochastic differential equations

$$a(D)Y(t) = b(D)DL(t),$$

where D denotes differentiation with respect to t,

$$a(z) := z^p + a_1 z^{p-1} + \cdots + a_p, \qquad (1)$$

$$b(z) := b_0 + b_1 z + \cdots + b_{p-1}, \qquad (2)$$

$b_q = 1$ and $b_j = 0$ for $q < j < p$.

How to think about CARMA(2,1)?

We can represent $Y(t)$ as

$$Y(t) = \alpha_1 \int_{-\infty}^{t} e^{\lambda_1(t-s)} dL(s) + \alpha_2 \int_{-\infty}^{t} e^{\lambda_2(t-s)} dL(s),$$

where $\alpha_1 = \frac{b_0+\lambda_1}{\lambda_1-\lambda_2}$ and $\alpha_2 = \frac{b_0+\lambda_2}{\lambda_2-\lambda_1}$.

Looks familiar?

It's just a sum of two Lévy-driven OU processes!

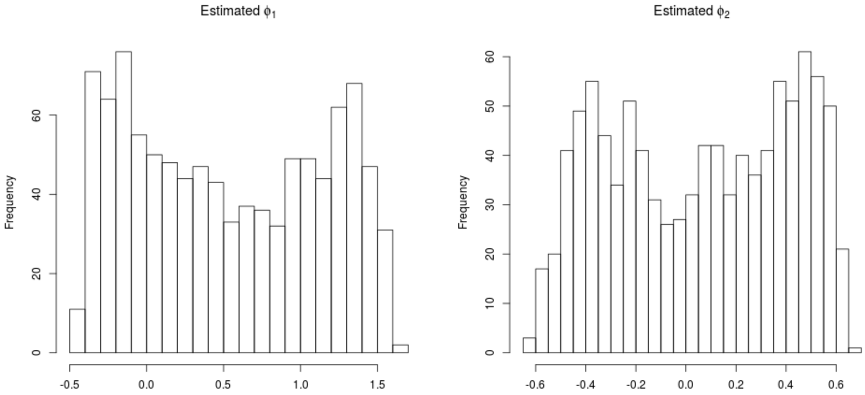
Fitted model

$$(D^2 + 0.847D + 0.122)Y(t) = (0.269 + D)DL(t)$$

	a_1	a_2	b_0
Estimates	0.847	0.122	0.269
Standard error	0.034	0.006	0.016
Relative error	0.019	0.015	0.031
Bias	0.639	0.211	0.096

Table: Estimated CARMA(2,1) parameters.

Houston, we have a problem!



Let's try a different method

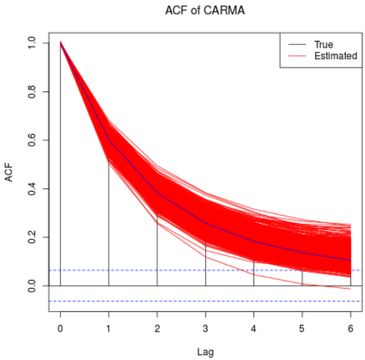


Figure: Autocorrelation functions of $Y(t)$: empirical (blue) and theoretical with parameters estimated from 1000 Monte Carlo simulations (red).

Who's guilty?

Fitting CARMA(2,1) model $\longleftrightarrow$ fitting a sum of two exponentials.

Acton [Act90]

... an exponential equation of this type [a weighted sum of two exponentials] in which all four parameters are to be fitted is extremely ill conditioned."

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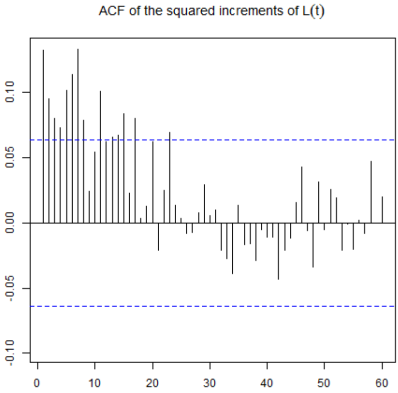
Is the recovered $L(t)$ really a Lévy process?



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Stochastic volatility

$$Y(t) = \int_{-\infty}^{t} g(t-s)\sigma_s dB_s,$$

where

$$\sigma_t^2 = \int_{-\infty}^{t} e^{-\delta(t-s)} dV_s,$$

$\delta > 0$, B_t is a standard Brownian motion and V_t – a Lévy subordinator independent of B_t.

Short memory OU-type volatility

Rough estimate: $\hat{\delta} = 1.82$ at significance level 0.01

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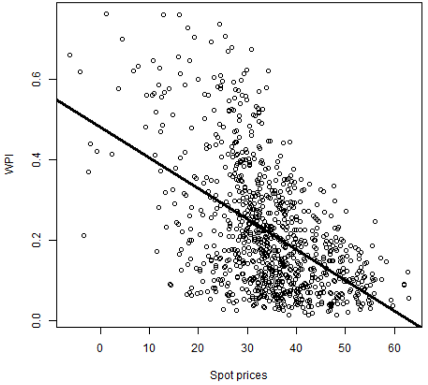
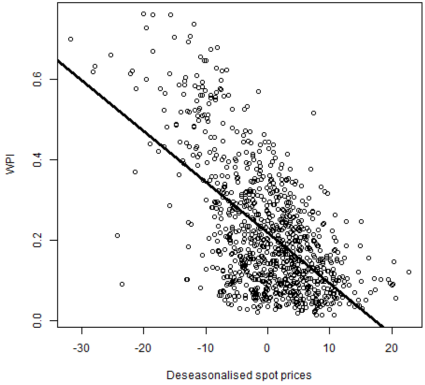
So what's blowing in the wind?





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Modified models

- $\Lambda(t)$ and $Z(t)$ unchanged;
- $Y(t)$: regress on wind data and fit CARMA(2,1) to the residuals.

$$Y_t = a_1 + a_2 \cdot \mathrm{WPI}_t + a_3 \cdot \mathrm{RD}_t + a_4 \cdot \mathrm{F}_t + a_5 \cdot \mathrm{WPI}_t^2 + a_6 \cdot \mathrm{RD}_t^2 + a_7 \cdot \mathrm{F}_t^2 + CARMA(2,1)$$

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Comparison of models

1. Simulate from the estimated process $\hat{Y}(t)$ 1000 times using the R package **yuima**.
2. Calculate the first n averaged moments ($n \in \{3, \ldots, 20\}$) of the simulated series.
3. Compute the sum of squared differences between "true" and simulated moments (*distance 1*).
4. Compute the sum of squared differences between "true" and simulated moments, normalised by the value of the appropriate true moment (*distance 2*).

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Winners and losers

- Best model: linear model with the residual demand $Y_t = a + b \cdot \mathrm{RD}_t + \mathrm{CARMA}_t$.
- Worst model: clearly the model without any wind data (so $Y_t = \mathrm{CARMA}_t$).

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Main contributions

- Allow the stochastic volatility in the short-term process;
- Include the influence of wind energy generation on the prices;
- Investigate problems related to CARMA(2,1) estimation.

What's next?

- Try different kernels (gamma?);
- Model wind energy generation directly;
- Study other types of renewables.

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Thank you for your attention!

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