Mean-reverting no-arbitrage additive models for forward curves in energy markets

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Energy Finance Christmas Workshop (EFC17)
13-15.12.2017, Kraków, Poland

Outline

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Power and gas forward markets

- Our purpose is to design tailor-made stochastic models for **power** and **gas** forward markets.
- Power and gas, differently from other commodities, are delivered as a given intensity over a certain time interval (e.g. a month, quarter or year).
- In financial terms, this means that energy forward contracts prescribe a **delivery period** for the underlying, rather than a maturity date.

- Price series exhibit **mean-reversion** and strong **seasonality**, which drives also prices' volatility.

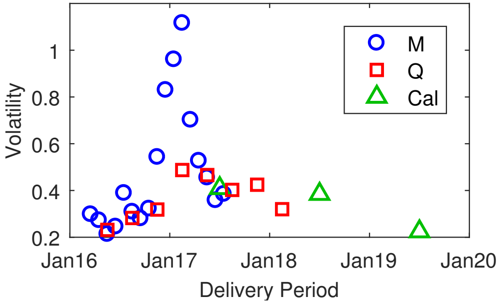

Figure: **Realized volatilities** of Phelix Base Futures (German power futures) from January 4, 2016 to May 23, 2017. The **Samuelson effect** (=volatility increasing as time approaches delivery) is not sufficient to explain this term structure.

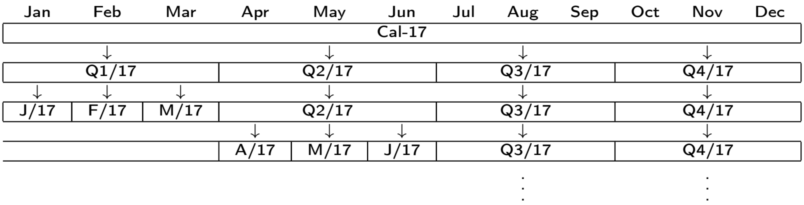

Figure: **Cascade unpacking mechanism** of forward contracts. For each given calendar year, as time passes forwards are unpacked first in quarters, then in the corresponding months. It may happen that the same delivery period is covered in the market by different contracts, e.g. one simultaneously finds quotes for Jan/17, Feb/17, Mar/17 and Q1/17.

Then, to avoid **arbitrage**, overlapping contracts must satisfy:

$$F(t, T_1, T_n) = \frac{1}{T_n - T_1} \sum_{i=1}^{n-1} (T_{i+1} - T_i) F(t, T_i, T_{i+1}). \qquad \text{(NA)}$$

Modeling framework

We want to introduce a stochastic dynamics for energy forward curves which is:

- **Realistic**
- **Tractable**
- **Arbitrage-free**

- We adopt the **Heath-Jarrow-Morton** methodology (vs. spot price based models), which consists in describing the whole forward curve by parametric SDEs [Benth and Koekebakker, 2008].
- The theoretical framework is constructed as follows. Let $f(t,T)$ denote the forward price at time t for **instantaneous delivery** at time T. Analogously, for any $T_1 < T_2$, let $F(t,T_1,T_2)$ be the forward price at time t for **delivery period** $[T_1,T_2]$.
- The forward prices evolve under the **real-world probability** measure $\mathbb{P}$ by

$$\begin{aligned} df(t,T) &= \big(-\lambda(t)f(t,T) + c(t,T)\big)\ dt + \theta(t,T)\ dW(t), \\ dF(t,T_1,T_2) &= \big(-\lambda(t)F(t,T_1,T_2) + C(t,T_1,T_2)\big)\ dt \\ &\quad + \Sigma(t,T_1,T_2)\ dW(t). \end{aligned}$$

- Under the physical measure we have a time dependent **mean-reversion** speed but NOT delivery dependent. The long term-mean and the diffusion coefficients (possibly many components) are both time and maturity dependent.
- Since we have not performed a logarithmic transformation of the price, the dynamics are **additive**. In principle, this could generate negative prices, but this is reasonable for power markets (given the presence of negative spot prices) and, for a suitable choice of coefficients, also for natural gas.
- These models are becoming more and more popular [Benth et al., 2007; Fanone et al., 2013; Kiesel and Paraschiv, 2017], since they generally allow to derive **analytical formulas** even for complex derivatives and risk measures.

- In power and gas markets the instantantaneous forwards do not exist. However, they play the role of **building blocks** for the dynamics of traded forwards:

$$F(t, T_1, T_2) = \frac{1}{T_2 - T_1} \int_{T_1}^{T_2} f(t, u) \, du,$$

  for all $t \leq T_1$ and $T_1 < T_2$.

- This relation implies **all** the no-arbitrage constraints of type (NA) (no static arbitrage).

- Dynamic arbitrage is prevented by constructing an **equivalent martingale measure** for the market, i.e. a probability $\mathbb{Q}$ equivalent to $\mathbb{P}$ such that

$$df(t,T) = \theta(t,T)\; dW^{\mathbb{Q}}(t),$$
$$dF(t,T_1,T_2) = \Sigma(t,T_1,T_2)\; dW^{\mathbb{Q}}(t),$$

  for some $\mathbb{Q}$-Brownian motion $W^{\mathbb{Q}}$.

- Generally NOT trivial at all (mean-reversion). For this reason we introduce the **key assumption**:

$$f(t,T) = \alpha(t,T)X(t) + \beta(t,T),$$

  where X is a universal source of randomness.

Generalized Lucia-Schwartz model

- This class of models is not void: the well-known **Lucia-Schwartz** model [Lucia and Schwartz, 2002] turns out to be of this kind:

$$
\begin{aligned}
df(t,T) &= \lambda(t)\big(\phi(T) - f(t,T)\big)\ dt \\
&\quad + e^{-\kappa(T-t)}\sigma_1\ dW_1(t) + \sigma_2\ dW_2(t), \\
dF(t,T_1,T_2) &= \lambda(t)\left(\frac{1}{T_2 - T_1}\int_{T_1}^{T_2} \phi(T)\ dT - F(t,T_1,T_2)\right) dt \\
&\quad + \sigma_1\, e^{\kappa t}\, \frac{(e^{-\kappa T_1} - e^{-\kappa T_2})}{\kappa(T_2 - T_1)}\, dW_1(t) + \sigma_2\, dW_2(t).
\end{aligned}
$$

- However, the volatility succeeds only in reproducing the Samuelson effect as an exponential decay (**no finer term structures**).

- We propose a modification of the Lucia-Schwartz model such that both price level and volatility are allowed to have a **non-trivial term structure**:

$$df(t,T) = \lambda(t)\big(\phi(T) - f(t,T)\big)\, dt + e^{-\kappa(T-t)}\sigma_1\, dW_1(t) + \psi(T)\, dW_2(t),$$

$$dF(t,T_1,T_2) = \lambda(t)\big(\Phi(T_1,T_2) - F(t,T_1,T_2)\big)\, dt + e^{\kappa t}\,\Gamma(T_1,T_2)\, dW_1(t) + \Psi(T_1,T_2)\, dW_2(t),$$

where

$$\Phi(T_1,T_2) := \frac{1}{T_2 - T_1}\int_{T_1}^{T_2} \phi(u)\, du$$

$$\Gamma(T_1,T_2) := \frac{1}{T_2 - T_1}\int_{T_1}^{T_2} \sigma_1 e^{-\kappa u}\, du = \frac{\sigma_1(e^{-\kappa T_1} - e^{-\kappa T_2})}{\kappa(T_2 - T_1)},$$

$$\Psi(T_1,T_2) := \frac{1}{T_2 - T_1}\int_{T_1}^{T_2} \psi(u)\, du.$$

- We have a **Samuelson effect** in both the instantanteneous (non-traded) forward prices $f(\cdot, T)$ and the traded forward prices $F(\cdot, T_1, T_2)$.
- The second factor describes the **volatility term structure** in an arbitrage-free way.
- Forward prices with shorter delivery periods are **more volatile** than forward prices with longer delivery periods. For instance, being equal the time to maturity, monthly contracts are more volatile than quarters (3 months) or calendars (1 year).

Estimation technique

- Estimation is performed **directly** on the market time series of traded forwards $F(\cdot, T_1, T_2)$ and no forward curve smoothing procedure is needed.
- For the **volatility** this task is not trivial, since the term structure is also maturity-dependent.
- We apply a method [Edoli et al., 2014] based on **quadratic variation/covariation** of price processes, which allows to estimate the diffusion coefficients.

- Then, in order to estimate the mean-reversion speed and level, since the covariance matrix of joint returns is singular, we first calibrate the parameters on the **single contract**. Afterwards, we take into account the **no-arbitrage constraints** (NA) that must hold among overlapping contracts (e.g. 3 months / 1 quarter).
- This is done by combining maximum likelihood estimation (**MLE**) with **Lagrange multipliers**.
- For example, with the convention that $\Phi_{\mathrm{Q2/17}}$ denotes the parameter $\Phi(T_1, T_2)$ corresponding to the contract Q2/17, then

$$\Phi_{\mathrm{Q2/17}} = u_{\mathrm{Apr/17}}\Phi_{\mathrm{Apr/17}} + u_{\mathrm{May/17}}\Phi_{\mathrm{May/17}} + u_{\mathrm{Jun/17}}\Phi_{\mathrm{Jun/17}},$$

where the weights u_i are defined according to the number of days in the month/quarter (e.g. $u_{\mathrm{Apr/17}} = 30/91$).

Empirical results

- We apply this estimation technique on the **Phelix Base Futures** market. We consider all the daily closing prices of each monthly, quarterly and calendar forward contract traded from January 4, 2016 to May 23, 2017.
- We first estimate the diffusion coefficients by introducing in the second factor a **classical seasonal component**, which takes into account a possible non-seasonal long-run volatility trend.
- Then, we compare it to a **nonparametric volatility shape**, where the second factor is a free parameter for each "atomic" forward.

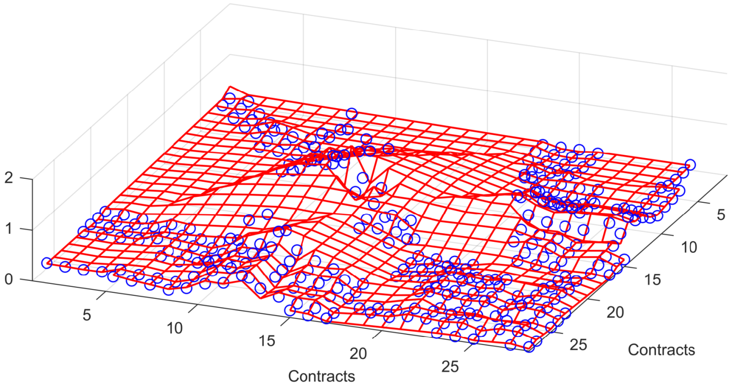

Figure: Historical and theoretical (nonparametric) covolatilities of all the contracts.

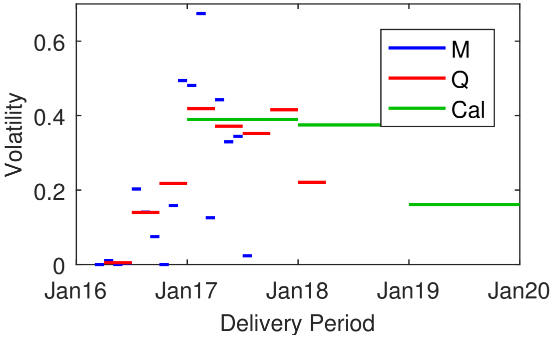

Figure: Estimated parameter Ψ for the second diffusion component of each contract.

Simulation study

- We then do a simulation study and assess the **performance** of the model.
- Firstly, we compare **simulated paths** of some exemplary futures contracts to the corresponding **observed trajectories**, so to discuss the qualitative behavior of model simulations.
- Secondly, we **compute fundamental statistics** of the model by averaging the results of a set of simulations and compare them to our data.

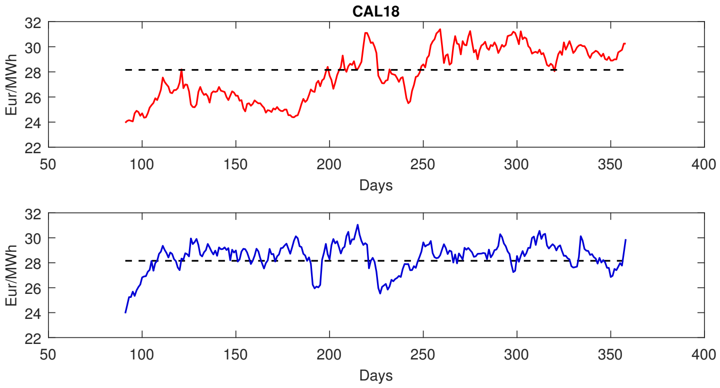

Figure: Historical (red) and simulated (blue) path of the contract Cal-18. The dotted line represents in both plots the estimated long-term mean of the contract.

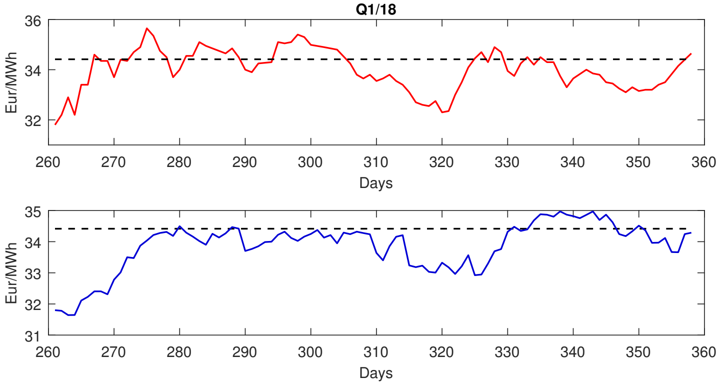

Figure: Historical (red) and simulated (blue) path of the contract Q1/18. The dotted line represents in both plots the estimated long-term mean of the futures price.

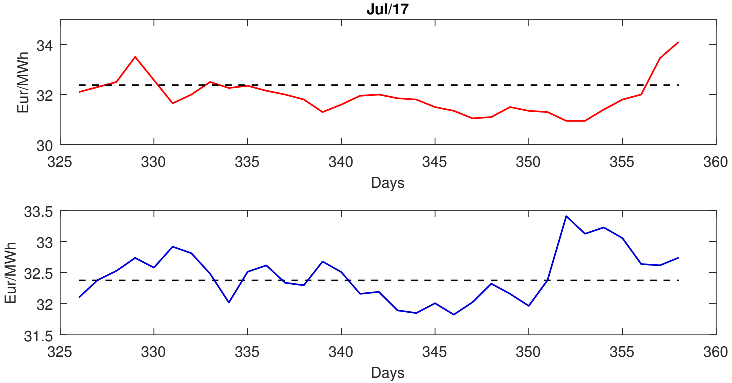

Figure: Historical (red) and simulated (blue) path of the contract Jul/17. The dotted line represents in both plots the estimated long-term mean of the futures price.

- For a more rigorous discussion of the fitting quality, we investigate the statistical features of the model and make a comparison with the historical data.
- In the upcoming **table** we report the values of the first four moments, the minimum and the maximum of both **empirical and simulated returns**, i.e. the daily price increments, of all contracts. We run 1000 simulations and then average the results over all samples. The values are classified among different delivery periods in order to distinguish different behaviors among them.

	Mean	Std. Dev.	Skewness	Ex. Kurt.	Min.	Max.
Phelix Base (M)	0.04	0.58	0.25	0.83	−1.32	1.62
Phelix Base (Q)	0.02	0.44	0.19	1.17	−1.36	1.44
Phelix Base (Cal)	0.03	0.41	−0.18	0.74	−1.35	1.26
Model (M)	0.00	0.67	0.01	1.71	−1.86	1.88
Model (Q)	0.00	0.43	0.06	1.87	−1.44	1.47
Model (Cal)	0.02	0.39	0.08	1.13	−1.25	1.32
Phelix Base	0.03	0.52	0.19	0.92	−1.33	1.53
Model	0.00	0.57	0.03	1.69	−1.68	1.70

Table: Empirical vs. simulated statistics (first four normalized moments, minimum and maximum) of the daily returns of all the contracts aggregated by delivery period. Model statistics are computed by averaging the results of the estimators over 1000 simulations, first, for each contract and, second, among contracts grouped by delivery period. The last two rows show the overall results for all the contracts.

Final comments

- We have designed a modeling approach for energy forward curves capable to hold together **mean-reversion** and **arbitrage theory**.
- Within this framework, we have specified a **generalized Lucia-Schwartz** (additive) model, which is sufficiently **tractable** for calibration purposes.
- We have introduced an *ad hoc* estimation procedure which takes into account the **peculiar trading mechanism** of these markets and does not require an artificial curve construction.
- The model reproduces in a satisfactory way the **trajectorial** and **statistical** features of our dataset. In particular, in terms of moments, a Gaussian distribution seems reasonable.

Thank you for your attention!