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Modeling a non-linear impact of renewable energy forecasts on intra-day electricity prices

- Sergei Kulakov, Florian Ziel
  – University of Duisburg-Essen

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Basic motivation

- Growing dependency of energy sector on renewable resources
  - we place our focus solely on wind and solar power
    - its advantages are manifold
    - its supply is though sensitive to weather conditions
      - its generation size is thus certain only at the moment of physical delivery
- The market mechanism of EPEX SPOT exchange
  - prices for electricity are established multiple times a day
    - the classification below can hence be defined
      - day-ahead prices
        - are determined 24 hours before the physical delivery of electricity
      - intra-day prices
        - can be settled up until 30 minutes before the physical delivery of electricity
  - it follows that those prices are based on forecasts as to the renewable energy supply
    - naturally, intra-day predictions tend to be less erroneous
    - more accurate forecasts may drive the discrepancy between day-ahead and intra-day prices to narrow

Basic motivation: evidence from EPEX SPOT SE

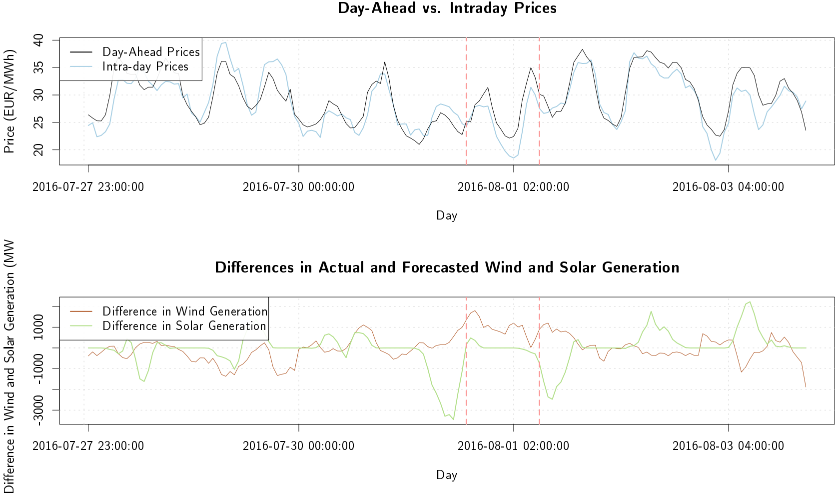


Figure: Dynamics of day-ahead and intra-day prices (upper graph) is plotted against the differences between realized and day-ahead forecasted wind and solar generation loads (lower graph) for a one-week sample from July, 27 to August, 03, 2016.

Our idea

- A forecasting error may exert a non-linear influence on the discrepancy between day-ahead and intra-day prices
  - we treat a forecasting error as a difference between realized and anticipated volumes of wind and solar power
  - the magnitude of the error's impact may depend on
    - the shape of the merit order curve
    - a sector of the merit order curve in which the market price is realized

Our idea: forecasted vs. realized supply curve

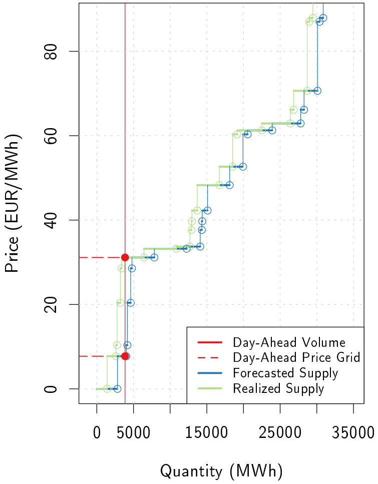


Figure: An example of a non-linear influence of a forecasting error on intra-day prices

Our idea (continued)

- We model intra-day prices given day-ahead data and the respective forecasting errors
  - empirical supply and demand curves provide a foundation for the modeling technique we used
    - the latter implies that our price can be determined as an equilibrium between a shifted supply and demand curves
    - an adjusted forecasting error determines the shift magnitude
      - we employ a non-linear optimization technique to define a necessary adjustment
- We want to prove that
  - a model which includes non-linear effects induced by a forecasting error may outperform a model which neglects them

Two benchmark models for the intra-day prices

- Naive

$$P_t^{naive} = P_t^{DA} + \varepsilon_t \tag{1}$$

  - where
    - P^{DA} stands for a day-ahead price
    - ε_t is an error term

- Linear 1

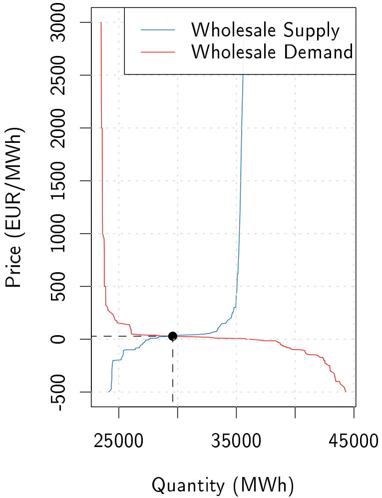
$$\begin{aligned} P_t^{lm_1} - P_t^{DA} = \beta_0 + \beta_1 \max(W_t^{\Delta}, 0) + \beta_2 \min(W_t^{\Delta}, 0) + \beta_3 \max(S_t^{\Delta}, 0) \\ + \beta_4 \min(S_t^{\Delta}, 0) + \beta_5 W_t^{A} + \beta_6 S_t^{A} + \varepsilon_t \end{aligned} \tag{2}$$

  - where
    - W_t^{Δ} is a wind forecasting error
    - S_t^{Δ} shows a solar forecasting error
    - W_t^{A} stands for an absolute amount of generated wind energy
    - S_t^{A} denotes an absolute volume of collected solar energy
  - note that we model positive and negative forecasting errors separately
    - this method was applied also in e.g. [Kiesel and Paraschiv, 2017], [Soysal et al., 2017], [Ziel, 2017]

- Linear 2

$$\begin{aligned} P_t^{lm_2} = \beta_0 + \beta_7 P_t^{DA} + \beta_1 \max(W_t^{\Delta}, 0) + \beta_2 \min(W_t^{\Delta}, 0) + \beta_3 \max(S_t^{\Delta}, 0) \\ + \beta_4 \min(S_t^{\Delta}, 0) + \beta_5 W_t^{A} + \beta_6 S_t^{A} + \varepsilon_t \end{aligned} \tag{3}$$

An example of empirical supply and demand curves



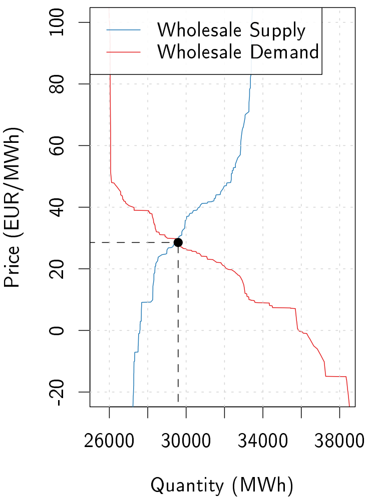
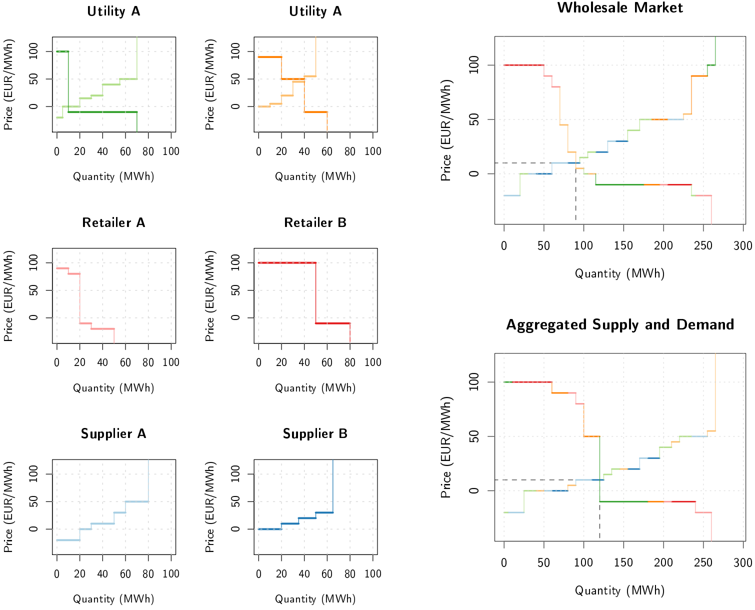


Figure: A wholesale market equilibrium on 2017-04-02 08-00-00 CET

An energy market: two perspectives [Knaut and Paulus, 2016]



Elastic demand curve vs. its inelastic analogue

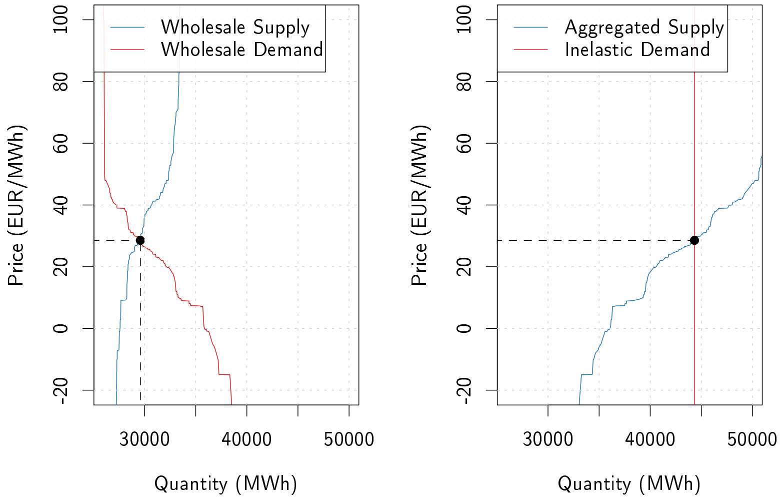


Figure: Wholesale market equilibrium on 2017-04-02 08-00-00 CET (left plot) vs. its manipulated form with an inelastic demand curve (right plot)

Transformation of supply and demand curves

- Transformation of the curves can be performed as follows
  - the formulas below were taken from [Coulon et al., 2014]
  - expression for an inelastic demand reads

$$Dem_t^{inelastic} = WSDem_t^{-1}(P_{\max}) \tag{4}$$

    - where
      - a demand curve in a wholesale market is abbreviated by $WSDem$
      - $P_{\max} = 3000$ as prescribed by the regulation of EPEX
  - equation for an inverse supply curve can be written as

$$Sup_t^{-1}(z) = WSSup_t^{-1}(z) + WSDem_t^{-1}(P_{\min}) - WSDem_t^{-1}(z) \tag{5}$$

    - where
      - a supply curve in a wholesale market is denoted by $WSSup$
      - $P_{\min} = -500$
    - note that the above equation defines $Sup_t(z)$ automatically
      - this holds since the function in question is monotonic

Our first model

- The model nlm_1 has the following specification
  - the expression for the shifted supply curve reads

$$Sup_t^{nlm_1}(z,\boldsymbol{\beta}_{nlm_1})=Sup_t\Big(z-\beta_8-\beta_9\max(W_t^{\Delta},0)-\beta_{10}\min(W_t^{\Delta},0) \\ -\beta_{11}\max(S_t^{\Delta},0)-\beta_{12}\min(S_t^{\Delta},0)-\beta_{13}W_t^{A}-\beta_{14}S_t^{A}\Big) \quad (6)$$

    - where
      - $\boldsymbol{\beta}_{nlm_1} = (\beta_8, ..., \beta_{14})$
      - W_t^{Δ} is a wind forecasting error
      - S_t^{Δ} shows a solar forecasting error
      - W_t^{A} stands for an absolute amount of generated wind energy
      - S_t^{A} denotes an absolute volume of collected solar energy
  - the intra-day price model is established as follows

$$P_t^{nlm_1}(\boldsymbol{\beta}_{nlm_1}) = Sup_t^{nlm_1}(Dem_t^{inelastic}, \boldsymbol{\beta}_{nlm_1}) + \varepsilon_t \quad (7)$$

    - where the first term on the right hand side represents an intersection between adjusted supply and inelastic demand curves
  - the vector of the coefficients $\boldsymbol{\beta}$ is estimated by solving the following non-linear least squares problem

$$\widehat{\boldsymbol{\beta}}_{nlm_1} = \underset{\boldsymbol{\beta}\in\mathbb{R}^7}{\arg\min}\left(P_t^{ID} - P_t^{nlm_1}(\beta_8, ..., \beta_{14})\right)^2 \quad (8)$$

    - R function `optim` was used as a major optimization tool

Our second model of intra-day prices

- The model nlm_2 aims to incorporate both linear and non-linear effects
  - the price equation of the model can thus be formulated as

$$P_t^{nlm_2}(\boldsymbol{\beta}_{nlm_2}) = \underbrace{P_t^{lm_2}(\beta_0, ..., \beta_7)}_{\text{linear component}} + \beta_{15} \underbrace{P_t^{nlm_1}(\beta_8, ..., \beta_{14})}_{\text{non-linear component}} + \varepsilon_t \quad (9)$$

    - note that
      - price in the linear model lm_2 is defined as

$$P_t^{lm_2}(\beta_0, ..., \beta_7) = \beta_0 + \beta_7 P_t^{DA} + \beta_1 \max(W_t^{\Delta}, 0) + \beta_2 \min(W_t^{\Delta}, 0) + \beta_3 \max(S_t^{\Delta}, 0) + \beta_4 \min(S_t^{\Delta}, 0) + \beta_5 W_t^A + \beta_6 S_t^A \quad (10)$$

      - price in the non-linear model nlm_1 is expressed as

$$P_t^{nlm_1}(\beta_8, ..., \beta_{14}) = Sup_t^{nlm_1}(Dem_t^{inelastic}, \boldsymbol{\beta}_{nlm_1}) \quad (11)$$

    - it follows that the model depends on the vector $\boldsymbol{\beta}_{nlm_2} = (\beta_0, ..., \beta_{15})$
  - writing the respective non-linear least squares problem yields

$$\widehat{\boldsymbol{\beta}}_{nlm_2} = \arg\min_{\beta \in \mathbb{R}^m} \left(P_t^{ID} - P_t^{nlm_2}(\beta_0, ..., \beta_{15})\right)^2 \quad (12)$$

    - the initial values for the coefficients $(\beta_0, ..., \beta_{14})$ were taken from the models lm_2 and nlm_1

Model comparison

- The obtained β-coefficients for the year 2016 are summarized in the table below

	Multiplier	lm	lm_2	nlm_1	nlm_2
β_0	–	78.77	-240.53	–	-4.59
β_1	$\max(W_t^{\Delta}, 0)$	-0.96	-0.97	–	-0.305
β_2	$\min(W_t^{\Delta}, 0)$	-1.04	-1.03	–	-0.075
β_3	$\max(S_t^{\Delta}, 0)$	-0.77	-0.81	–	-0.559
β_4	$\min(W_t^{\Delta}, 0)$	-1.46	-1.44	–	-0.065
β_5	W_t^A	0.03	0.037	–	-0.057
β_6	S_t^A	0.004	0.0090	–	-0.613
β_7	P_t^{DA}	1	1.00085	–	0.652
β_8	–	–	–	0.10	-0.039
β_9	$\max(W_t^{\Delta}, 0)$	–	–	0.35	-0.090
β_{10}	$\min(W_t^{\Delta}, 0)$	–	–	0.24	0.256
β_{11}	$\max(S_t^{\Delta}, 0)$	–	–	0.62	0.552
β_{12}	$\min(W_t^{\Delta}, 0)$	–	–	0.31	0.332
β_{13}	W_t^A	–	–	-0.018	0.547
β_{14}	S_t^A	–	–	-0.023	0.68
β_{15}	$P_t^{nlm_1}$	–	–	–	0.369

Model comparison

- The out-of-sample performance of the models can be described as follows

	MAE	RMSE
Naive	5.053	8.731
lm_1	4.495	**8.160**
lm_2	4.519	8.222
nlm_1	4.561	8.825
nlm_2	**4.473**	8.242

  - both MAE and RMSE tests were conducted using a rolling time window
    - the number of in-sample observations equals to 365 days
      - year 2016 was taken as an initial time frame
    - the out-of-sample horizon is limited to 186 days
    - the window size is 24 hours
- The obtained results are in general ambiguous
  - linear model lm_2 fails to surpass the model lm_1
  - model lm_1 produces lesser MAE and RMSE errors than the model nlm_1
  - model nlm_2 bears the smallest MAE figure and second lowest RMSE value

Model comparison

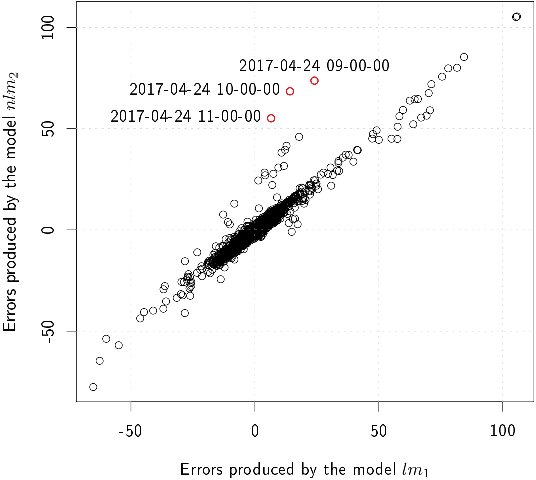
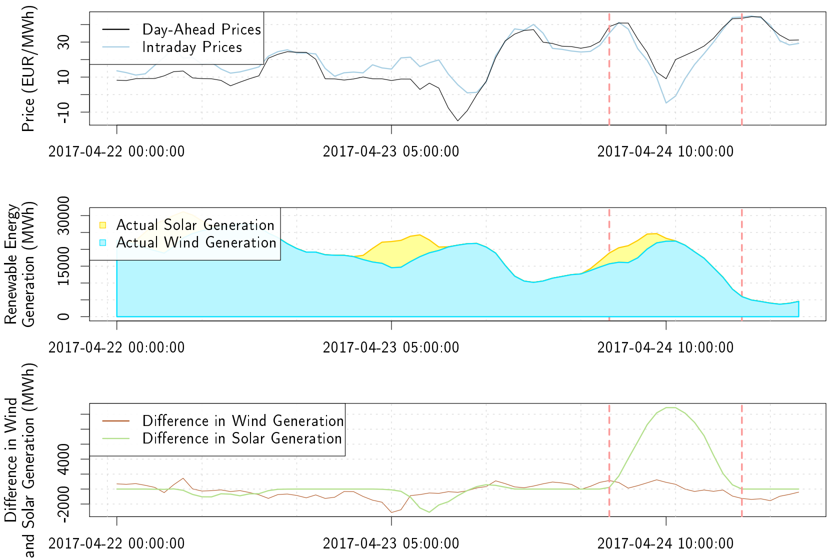


Figure: Errors of the models lm_1 (x-axis) and nlm_2 (y-axis) computed as an absolute difference between the observed and the suggested-by-the-models intra-day prices for the out-of-sample time span

Model comparison: a possible nature of the outliers



Concluding remarks

- By taking advantage of the empirical supply and demand curves we showed that
  - equilibrium in wholesale market coincides with that of aggregated supply and demand curves
    - it follows that both equilibria yield identical price
  - it is possible to model intra-day prices given the day-ahead data and forecasting errors in wind and solar power
  - the accuracy of a model improves whenever it encompasses non-linear effects implied by the errors
- Steps to be undertaken
  - the outliers are to be given a more thorough investigation
  - the model nlm_2 is to be tested without parameter $\beta_7 P^{DA}$
    - this point stems from the fact that the model lm_1 is more accurate than the model lm_2
  - a more elaborated optimization tool can be employed

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