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Modeling market order arrivals on the intraday power market for deliveries in Germany with Hawkes processes with parametric kernels

Energy Finance Christmas Workshop

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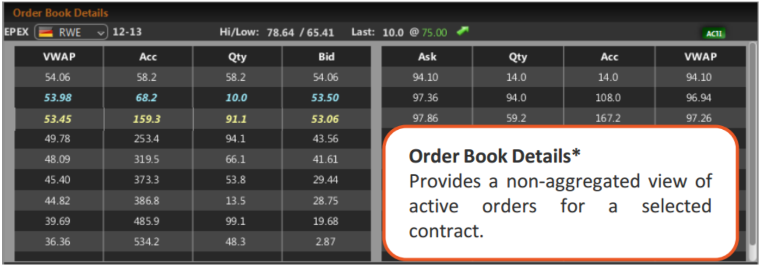
Motivation

- Literature on empirical insights into characteristics of the intraday power market for deliveries in GER evolving from continuous trading is currently rather scarce.
- By using a comprehensive dataset of limit and market orders, we aim at providing further empirical insights into how trading on the market evolves.

Limit Order Book

- Sell and buy limit orders (LO)
- Limit order book (LOB) visible to all market participants
- Bid-ask spread is difference between best sell and buy LO, here 40.04 EUR per MW (extremely high)



Order Book Details

EPEX RWE 12-13 Hi/Low: 78.64 / 65.41 Last: 10.0 @ 75.00

VWAP	Acc	Qty	Bid	Ask	Qty	Acc	VWAP
54.06	58.2	58.2	54.06	94.10	14.0	14.0	94.10
53.98	68.2	10.0	53.50	97.36	94.0	108.0	96.94
53.45	159.3	91.1	53.06	97.86	59.2	167.2	97.26
49.78	253.4	94.1	43.56				
48.09	319.5	66.1	41.61				
45.40	373.3	53.8	29.44				
44.82	386.8	13.5	28.75				
39.69	485.9	99.1	19.68				
36.36	534.2	48.3	2.87				

Order Book Details*
Provides a non-aggregated view of active orders for a selected contract.

Source: `https://www.epexspot.com/document/30313/ComTrader%20-%20Guideline`

Figure: Snapshot of limit order book for product H13.

Orders

- A buy (sell) limit order (LO) is an instrument which allows an agent to express how much she wants to pay (receive) per share for a specific number of shares.
- All unfilled buy and sell limit orders are gathered in the limit-order book (LOB).
- A buy (sell) market order (MO) is an instrument which allows an agent to buy (sell) a specific number of shares at the current best sell (buy) limit order price(s).

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Spot power market for deliveries in GER

- Contracts with hourly and quarter hourly delivery; since end of March 2017 also half hourly contracts.
- For hourly contracts, there is a day-ahead auction and continuous trading until half an hour before delivery start:

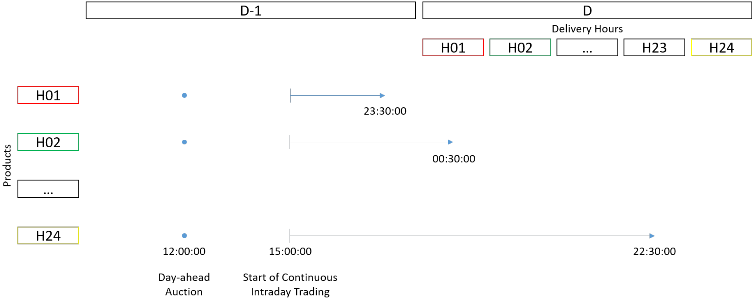

Figure: Schema of GER spot power market.

Structure of GER intraday power market

- There is central limit order book containing market participants' limit orders (LOs) for deliveries in GER; these can be filled by placing market orders (MOs).
- Iceberg orders are a special type of LO which enter into order book. They have a peak quantity visible to the market and a hidden quantity.
- An important feature of the market is that LOs from other delivery areas appear in the order book for deliveries in GER if interconnector capacity is available and vice versa.

Data

- We use data comprising all LOs and MOs which had GER as delivery area. That means that e.g. LOs which had FRA as delivery area are not contained.
- We consider two time periods: Q2/2015 and Q2/2016; important structural differences are (i) that the leadtime decreased from 45 mins to 30 mins and (ii) that the price tick size was increased from 0.01 to 0.1 EUR per MWh.
- We remove data for weekend delivery days.

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LO arrivals per trading day

Q2/2015									
	Buy					Sell			
	Mean	SD	Skew	Med		Mean	SD	Skew	Med
H04	218.2	69.9	1.35	191.0	H04	220.8	82.3	1.65	197.0
H10	491.7	324.8	4.58	423.0	H10	513.5	597.7	7.30	417.0
H16	699.3	447.1	4.21	586.0	H16	650.8	484.4	6.39	581.0
H22	575.7	582.3	4.93	424.0	H22	513.7	432.2	4.51	417.0
Q2/2016									
	Buy					Sell			
	Mean	SD	Skew	Med		Mean	SD	Skew	Med
H04	509.1	165.9	0.98	484.0	H04	465.9	160.6	0.94	431.0
H10	859.2	292.0	1.72	790.0	H10	769.5	198.7	0.64	748.0
H16	1,073.7	319.7	1.48	1,025.0	H16	1,007.4	312.4	2.25	997.0
H22	800.4	274.8	1.10	739.0	H22	689.0	179.6	0.53	676.0

Table: Descriptive statistics on LO arrivals per trading day.

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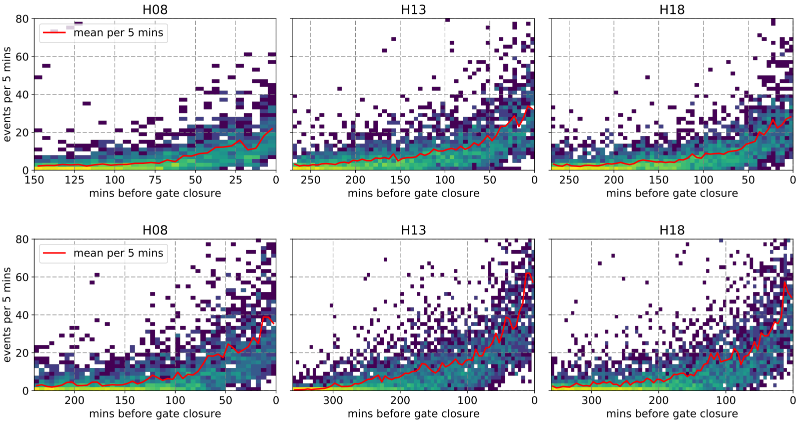

Figure: Distributions of buy LO arrivals per 5 mins and means for Q2/2015 (upper) and Q2/2016 (lower).

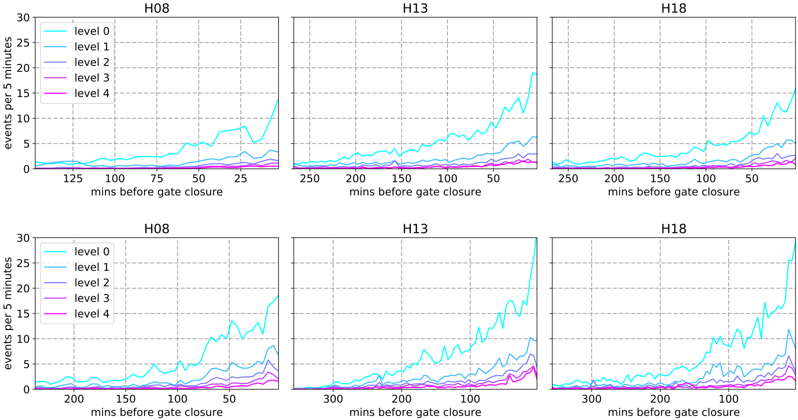

Figure: Mean numbers of buy LO arrivals on the first five order book levels per 5 mins for Q2/2015 (upper) and Q2/2016 (lower).

Price differences per level

Q2/2015											
	Buy						Sell				
	0	1	2	3	4		0	1	2	3	4
H08	-0.72	0.78	1.68	3.09	4.81	H08	-0.74	0.56	1.43	2.49	3.88
H13	-0.37	0.39	0.84	1.45	1.85	H13	-0.37	0.37	0.85	1.74	2.21
H18	-0.36	0.32	0.71	1.22	1.57	H18	-0.45	0.41	0.90	1.46	2.12
Q2/2016											
	Buy						Sell				
	0	1	2	3	4		0	1	2	3	4
H08	-0.42	0.41	0.86	1.28	2.06	H08	-0.44	0.34	0.74	1.33	2.17
H13	-0.25	0.20	0.42	0.68	0.98	H13	-0.27	0.21	0.45	0.68	1.03
H18	-0.20	0.16	0.36	0.62	0.98	H18	-0.28	0.22	0.45	0.70	1.05

Table: Mean differences b/w prices of limit orders placed on level $0, \ldots, 4$ and best bid/ask prices before placement.

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Share per no. of prices executed against

	Q2/2015								
	Buy					Sell			
	1	2	3	4		1	2	3	4
H08	83.7%	10.8%	3.7%	1.0%	H08	84.6%	10.5%	3.4%	0.7%
H13	80.8%	11.9%	4.2%	1.6%	H13	84.1%	10.4%	3.3%	1.3%
H18	83.7%	10.8%	3.3%	1.4%	H18	82.2%	11.5%	4.1%	1.3%
	Q2/2016								
	Buy					Sell			
	1	2	3	4		1	2	3	4
H08	75.7%	13.0%	6.4%	2.9%	H08	79.2%	11.4%	5.2%	2.6%
H13	80.4%	11.8%	4.6%	1.9%	H13	78.8%	12.4%	5.4%	2.0%
H18	82.2%	11.1%	4.1%	1.7%	H18	79.3%	12.7%	4.7%	2.0%

Table: Shares of MOs executed against $1, \dots, 4$ price/s in all MOs.

Execution costs per no. of prices executed against

	Q2/2015								
	Buy					Sell			
	1	2	3	4		1	2	3	4
H08	0.00	0.27	0.61	0.61	H08	0.00	0.47	0.63	0.81
H13	0.00	0.38	0.76	0.83	H13	0.00	0.18	0.23	0.33
H18	0.00	0.13	0.26	0.43	H18	0.00	0.12	0.28	0.38
	Q2/2016								
	Buy					Sell			
	1	2	3	4		1	2	3	4
H08	0.00	0.13	0.25	0.46	H08	0.00	0.12	0.25	0.38
H13	0.00	0.09	0.20	0.29	H13	0.00	0.09	0.17	0.26
H18	0.00	0.08	0.18	0.31	H18	0.00	0.07	0.16	0.27

Table: Mean execution costs of MOs executed against $1, \ldots, 4$ price/s.

Bid-ask spreads

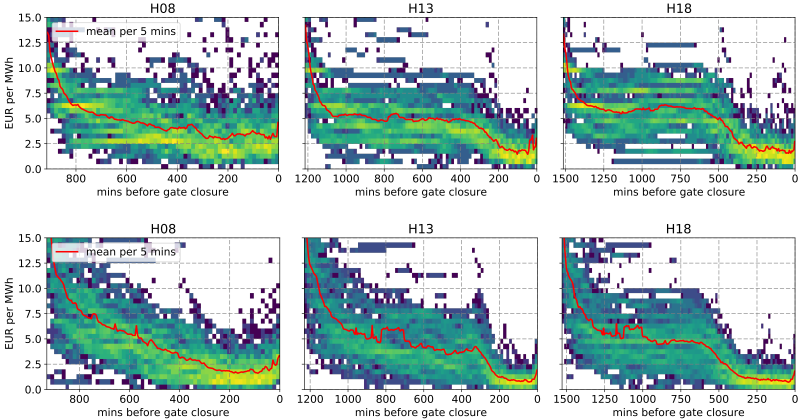

Figure: Distributions of 5-minute time-weighted bid-ask spreads and means in for Q2/2015 (upper) and Q2/2016 (lower).

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Empirics I

- Shape of distributions of LO arrivals per trading day depends on delivery hour. We observe positive skew. Average numbers have increased between Q2/2015 and Q2/2016. Similar for LO cancellations and MO arrivals.
- In the hours before gate closure average LO arrivals increase more than linearly. The same holds true for LO cancellations and MO arrivals.

Empirics II

- Average buy LO arrivals on level 0 per 5 mins exceed those on other levels.
- In Q2/2015, average buy LO cancellations from level 0 are close to those from level 1; in Q2/2016, they are close to each other.
- Average buy LO arrivals on level 0 exceed average buy LO cancellations from level 0.
- In Q2/2015, average amounts by which buy LO arrivals on level 0 decrease bid-ask spread are below average amounts by which buy LO cancellations from level 0 increase bid-ask spread; in Q2/2016 they are closer to each other.

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Why modeling market order arrivals?

- When studying markets from an intraday perspective, the times when market orders arrive play a key role.
- One reason for this is that market orders may change the mid price and bid-ask spread.
- Algorithmic trading strategies may be designed such that the nature of market orders including the times when they arrive have an impact on the optimal behavior.

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Buy MO arrivals

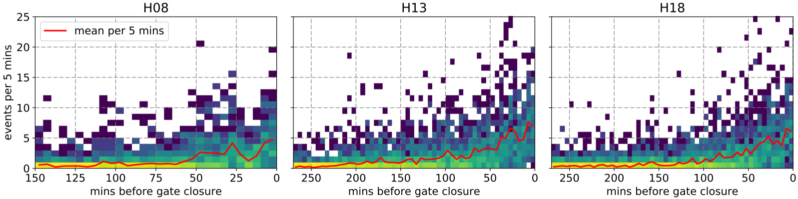

Figure: Distributions of buy MO arrivals per 5 mins and means for Q2/2015.

Test for non-homogeneous Poisson

- The test requires specifying a time interval L during which the arrivals may be considered to have a constant rate.
- The timestamps of all arrival events contained in time bin i, $T_{i,j}$ with $j \in \{1, \ldots, J(i)\}$, are transformed according to

$$R_{i,j} = (J(i) + j - 1)\left(-\log\left(\frac{L - T_{i,j}}{L - T_{i,j-1}}\right)\right).$$

- If the arrival rate is indeed constant per specified time interval, the $\left\{R_{i,j}\right\}$ are standard exponentially distributed.

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Test results

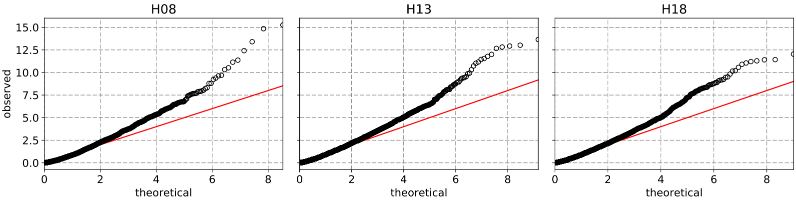

Figure: Q2/2015.

Figure: QQ plots of transformed timestamps of buy MO arrivals and the unit-rate exponential distribution for Q2/2015.

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Standard Hawkes process

Definition (Standard Hawkes Process)

Let $\{N(t), t \geq 0\}$ be counting process. It is said to be a standard Hawkes process if its intensity has the form

$$\begin{aligned}\lambda(t) &= \mu(t) + \int_{-\infty}^{t} \phi(t-u)dN_u \\ &= \mu(t) + \sum_{t_i<t} \phi\left(t - t_i\right),\end{aligned}$$

where $\mu\colon \mathbb{R} \mapsto \mathbb{R}_{>0}$ is the deterministic baseline intensity and $\phi\colon \mathbb{R}_{\geq 0} \mapsto \mathbb{R}_{\geq 0}$ maps the impact of past events on the intensity and may be referred to as excitement function.

Goodness-of-fit

Theorem (Time Change)

Let N be a simple point process adapted to a history $\mathcal{F}$ with bounded, strictly positive conditional $\mathcal{F}$-intensity $\lambda(t)$ and $\mathcal{F}$-compensator $\Lambda(t) = \int_0^t \lambda(u)du$ that is not a.s.-bounded. Under the random time change $t \mapsto \Lambda(t)$, the transformed process

$$\tilde{N}(t) = N(\Lambda^{-1}(t))$$

is a Poisson process with unit rate.

Test for unit-rate exponential distribution: Kolmogorov-Smirnov
Test for independence: Ljung-Box

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One-kernel models

Consider a one-kernel Hawkes process

$$\lambda(t) = \mu(t) + \sum_{t_i < t} \phi\left(t - t_i\right),$$

where baseline intensity is

$$\mu(t) = \gamma e^{\delta t}.$$

We consider exponential excitement function, i.e.

$$\phi(t) = \alpha e^{-\beta t}.$$

Estimation

Proposition (Log Likelihood)

The log-likelihood of a exponential-kernel Hawkes process can be explicitly computed:

$$\begin{aligned} \log L\left(\{t_i\}_{i=1,\dots,n}\right) &= t_n - \int_0^{t_n} \mu(s)ds - \sum_{i=1}^{n} \frac{\alpha}{\beta}\left(1 - e^{-\beta(t_n - t_i)}\right) \\ &\quad - \sum_{i=2}^{n} \log\left[\mu(t_i) + \alpha R(i)\right], \end{aligned}$$

where $R(.)$ is recursively defined with $R(1) = 0$ and

$$R(i) = e^{-\beta(t_i - t_{i-1})}\left(1 + R(i-1)\right)$$

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Goodness-of-fit

	Exponential			
	N_E	$p_{KS} > 0.05$	$p_{LB}^{L1} > 0.05$	$p_{LB}^{L2} > 0.05$
H00	91	90	84	86
H02	91	90	87	87
H04	91	91	85	84
H06	91	89	86	89
H08	91	90	87	89
H09	91	89	78	78
H10	91	89	80	83
H11	91	88	86	84
H12	91	91	84	82
H13	91	88	81	82
H14	91	88	82	81
H15	91	91	86	86
H16	91	91	81	84
H18	91	87	81	82
H20	91	89	73	76

Table: Number of times KS and LB null hypotheses are not rejected at 5% level.

Branching ratios

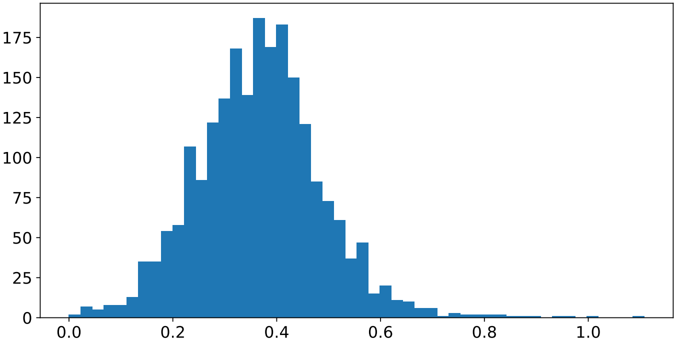

Figure: Histogram of branching ratios from model with exponential excitement (α/β).

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Different types of market orders

- We investigate the nature of market orders.
- Examples of market orders of different nature are:
  (i) One observes clusters of market orders which are executed against iceberg orders. These clusters should be considered separately as they don't have an impact on the bid-ask spread (as opposed to regular market orders).
  (ii) To keep their order-to-trade ratio below the limit market participants may send market orders of minimum size (i.e. 0.1 MW) into the market. One may want to consider this type of action separately.

Two-kernel models

Let $\{t_i^0, i \geq 0\}$ denote a sequence of market order arrival times with volume 25 MW and $\{t_i^1, i \geq 0\}$ a sequence of market order arrival times with volumes other than 25 MW. We estimate the intensities

$$\lambda_0(t) = \mu_0(t) + \sum_{t_i^0 < t} \phi_{00}(t - t_i^0) + \sum_{t_i^1 < t} \phi_{10}(t - t_i^1)$$
$$\lambda_1(t) = \mu_1(t) + \sum_{t_i^1 < t} \phi_{11}(t - t_i^1) + \sum_{t_i^0 < t} \phi_{01}(t - t_i^0),$$

with baseline intensity and excitement as before.

Model selection

- A natural question is whether we gain anything by splitting up the market orders or loose something (bias-variance tradeoff).
- Furthermore, one may ask whether both variates should have an impact on themselves and the other variate or whether other constellations are more suited (e.g. the first variate is impacted by itself and the second whereas the second is only impacted by the first.
- We cannot simply consult AIC due to the fact that we estimate on the basis of two different datasets.
- To overcome this we use the following algorithm to generate mean squared errors for for each potential model which we then compare.

Model selection cont'd

Algorithm 1 Computation of mean squared errors

procedure COMPUTEMSE($start$, T, $interval$)
  $T_{int} \leftarrow start$
  while $T_{int} < T$ **do**
    for $i \leftarrow 1$, number of models **do**
      estimate and assess goodness-of-fit model i
    end for
    for $i \leftarrow 1$, number of model combinations **do**
      for $j \leftarrow 1$, number of simulations **do**
        simulate number of market orders until T
      end for
      compute mean number of market orders until T
      compute squared error
    end for
    $T_{int} = T_{int} + interval$
  end while
  for $i \leftarrow 1$, number of model combinations **do**
    compute mean squared error
  end for
end procedure

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Model selection cont'd

- For simulation we use the method proposed in Chen and Stindl (forthcoming) extended to multivariate and by allowing for non-zero start time.
- Additional advantage is that it is possible to compare to autoregressive conditional duration (ACD) models.

Goodness-of-fit

We consider split between 25 MW market orders and all other market orders.

Variate	Covariate/s	N	$N_{p_{KS}>0.05}$	$N_{p_{LB}>0.05}$
0	0	47	47	44
0	1	47	47	44
0	0, 1	47	47	44
1	0	47	0	40
1	1	46	46	46
1	0, 1	45	45	26

Table: Number of times KS null hypothesis is not rejected at 5% level and LB null hypothesis is not rejected at 5% level for five lags at most for all models estimated for the two variates and delivery start 2015-04-20 12:00:00 UTC.

Comparison of MSE

Covariate of Variate 0	Covariate of Variate 1	MSE
0	0	53.401
0	1	18.475
0	0, 1	82.921
1	0	45.771
1	1	16.367
1	0, 1	72.473
0, 1	0	56.197
0, 1	1	18.249
0, 1	0, 1	89.277

Table: Mean squared errors for all combinations of the models for the two variates and delivery start 2015-04-20 12:00:00 UTC.

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Conclusion and Outlook

- Hawkes process with exponential baseline intensity and exponential excitement function seems to be able capture the dynamics of market order arrivals well.
- Apart from two models goodness-of-fit looks good.
- Mean squared errors indicate that models where variate 1 (i.e. rest) has only itself as covariate are the best ones.
- Variation in squared error is expected to be decreasing with decreasing time-to-maturity; we haven't thought through yet whether this is undesireable.
- Question whether multivariate models outperform the univariate over multiple days and delivery hours is yet to be answered. Pattern?

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