Mean Field Game and local storages in the power system

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joint work with I. Ben Tahar [2], Anis Matoussi[3]

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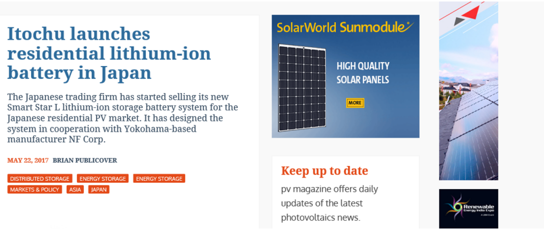
Motivations



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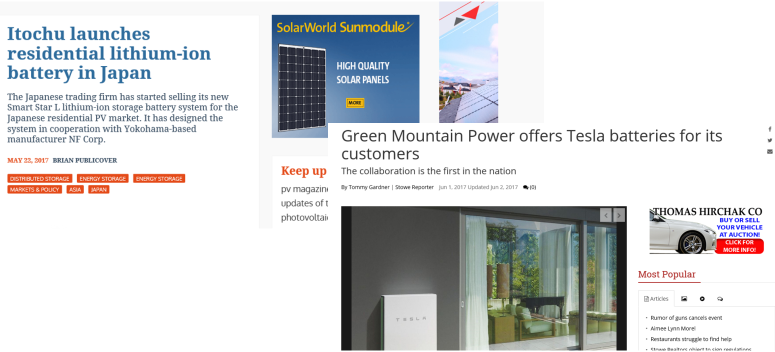

Itochu launches residential lithium-ion battery in Japan

The Japanese trading firm has started selling its new Smart Star L lithium-ion storage battery system for the Japanese residential PV market. It has designed the system in cooperation with Yokohama-based manufacturer NF Corp.

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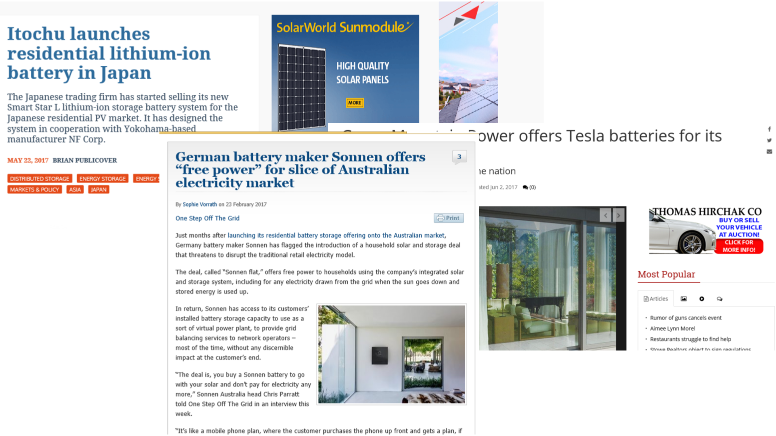

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German battery maker Sonnen offers "free power" for slice of Australian electricity market

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By Sophie Vorrath on 23 February 2017

One Step Off The Grid

Print

Just months after launching its residential battery storage offering onto the Australian market, Germany battery maker Sonnen has flagged the introduction of a household solar and storage deal that threatens to disrupt the traditional retail electricity model.

The deal, called "Sonnen flat," offers free power to households using the company's integrated solar and storage system, including for any electricity drawn from the grid when the sun goes down and stored energy is used up.

In return, Sonnen has access to its customers' installed battery storage capacity to use as a sort of virtual power plant, to provide grid balancing services to network operators – most of the time, without any discernible impact at the customer's end.

"The deal is, you buy a Sonnen battery to go with your solar and don't pay for electricity any more," Sonnen Australia head Chris Parratt told One Step Off The Grid in an interview this week.

"It's like a mobile phone plan, where the customer purchases the phone up front and gets a plan, if

...ower offers Tesla batteries for its

...e nation

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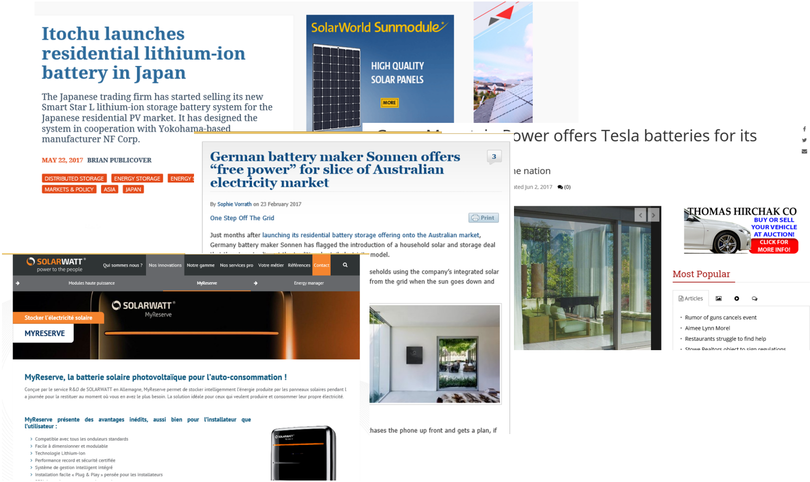

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Technologie Lithium-Ion
Performance record et sécurité certifiée
Système de gestion intelligent intégré
Installation facile « Plug & Play » pensée pour les installateurs

Context

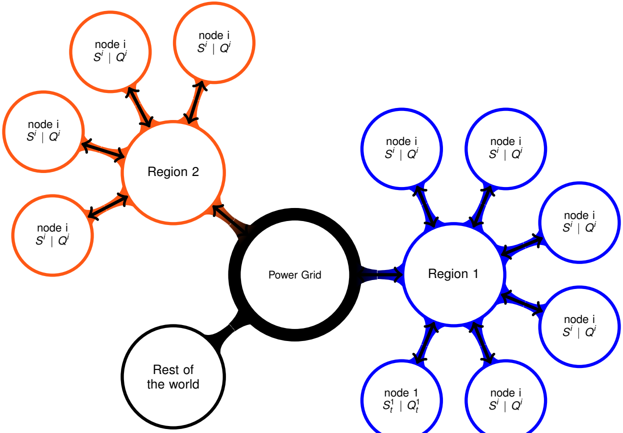
- Willingness of more local control/decision on electricity production/consumption
- Electricity storage triggered by the expansion of geographically (large to small scale) distributed generation

Goal: characterize the storage management and sizing of decentralized storages taking into account the

- Impact of energy and distribution tariff structure
- Impact of level of development of decentralized storage in the electricity system
- Competition/interaction with traditional players

- Extended MFG and MFG with common noise : R. Carmona and F. Delarue (2013), (2015), (2017); R. Carmona and F. Delarue and D. Lacker (2016), P. Cardialaguet and C.A. Lehalle (2017)
- Linear Quadratic case: A. Bensoussan, K. Sung, S. Yam and Yung (2011), Yong (2013), Graber (2016).
- Micro-storages in electrical system and MFG:
  R. Couillet, R., S. Medina Perlaza, H. Tembine, H. and M. Debbah (2012), A. de Paola, D. Angeli, and G. Strbac, G. (2016)

Overview of the model



- Grid = N nodes spread in Γ regions (N_γ Agents in node γ)
- S^i storage level of node i
- α_i storage rate of node i is the control of this node
- $(Q^i - \alpha_i)$ = power [injection if ≥ 0] / [consumption if ≤ 0]

- $B^0, B^1, \cdots, B^N$, N independent Brownian motions
- $\mathbb{F}^0 = \{\mathcal{F}_t^0\}$ the filtration generated by B^0, $\mathcal{F}_t^0 = \sigma(B_s^0,\ s \le t\}$

- "Rest of the world" power

$$dQ_t^r = \mu^\gamma(t, Q_t^r)dt + \sigma^{r0}(t, Q_t^r)dB_t^0\ ,\ \ Q_0^r = q_0^r \tag{1}$$

- Individual prosumers power in region γ

$$dQ_t^i = \mu^\gamma(t, Q_t^i)dt + \sigma^\gamma(t, Q_t^i)dB_t^i + \sigma^{\gamma 0}(t, Q_t^i)dB_t^0\ ,\ \ Q_0^i = q_0^i. \tag{2}$$

- Individual battery level,

$$S_t^{i,\alpha^i} = s_0^i + \int_0^t \alpha_s^i ds, \tag{3}$$

with $\alpha_i \in \mathcal{A}$, $\mathcal{A}$ the set of $\mathbb{F}$-adapted real-valued processes $a = \{a_t\}$ such that $\mathbb{E}\left[\int_0^T |a_u|^2 du\right] < \infty$

- Electricity price per Watt-hour, depends on all-players strategies:

$$P_t^{N,\alpha} = p\left(-Q_t^r - \sum_{i=1}^{N} \eta(Q_t^i - \alpha_t^i)\right)$$

- η is a scaling parameter such that $\eta \to 0$ and $N \to +\infty$ with the assumption that $\eta N \propto 1$ and $p(.)$ inverse demand function.

$$P_t^{N,\alpha} \simeq p\left(-Q_t^r - \sum_{i=1}^{N} \frac{1}{N}(Q_t^i - \alpha_t^i)\right)$$

By noting $\pi^\gamma = \frac{N_\gamma}{N}$

$$P_t^{N,\alpha} \simeq p\left(-Q_t^r - \sum_{\gamma=1}^{\Gamma} \pi^\gamma \sum_{i\in\gamma} \frac{1}{N_\gamma}(Q_t^i - \alpha_t^i)\right)$$

- Assumption: the function $p(.)$ is strictly increasing

Let $\alpha = (\alpha^1, \cdots, \alpha^N)$

- Cost function at node *i* in region γ:

$$J^{i,\gamma,N,\eta}(\alpha) = \mathbb{E}\int_0^T J^{\text{store}}(t, S_t^{i,\alpha^i}, \alpha_t^i) + J^{\text{demand charge}}(t, Q_t^i, \alpha_t^i)$$
$$+J^{\text{vol. charge}}(t, Q_t^i, \alpha_t^i, P_t^{N,\alpha})dt + \mathbb{E}[J^{\text{term. cost}(S_T^{i,\alpha^i})}]$$

$$J^{i,\gamma,N,\eta}(\alpha) = \mathbb{E}\int_0^T \Big[\frac{A}{2}|S_t^{i,\alpha^i}|^2 + L\,S_t^{i,\alpha^i} + \frac{C}{2}|\alpha_t^i|^2 + \frac{K^\gamma}{2}|Q_t^i - \alpha_t^i|^2$$
$$-P_t^{N,\alpha}\left(Q_t^i - \alpha_t^i\right)\Big]\,dt + \mathbb{E}[\tfrac{B}{2}(S_T^{i,\alpha^i} - b)^2]$$

- Egalitarian cost function for the Social planner

$$J^{\mathrm{C,N},\eta}(\alpha) = \mathbb{E}\left[\int_0^T \frac{K^\gamma}{2}\left|Q_t^r\right|^2 - P_t^{N,\alpha}Q_t^r dt\right] + \frac{1}{N}\sum_{\gamma=1}^{\Gamma}\sum_{i=1}^{N_\gamma} J^{i,\gamma,N,\eta}(\alpha)$$

Notation: for a $\mathbb{F}$-adapted process $\xi = \{\xi_t\}$, denote $\bar{\xi}^0_t := \mathbb{E}[\xi_t|\mathcal{F}^0_t]$

Let, - $\mathbb{F}^0$-adapted $\mathbb{R}^\Gamma$-valued process $\bar{\nu}^0 = \{(\bar{\nu}^{1,0}_t, \cdots, \bar{\nu}^{\Gamma,0}_t)\}$,
- Initial values: $x_0 = (s_0, q_0) = \{x^\ell_0 = (s^\ell_0, q^\ell_0), 1 \le \ell \le \Gamma\}$
- Control process $\alpha \in \mathcal{A}$, associate the cost

$$J^{\gamma,\mathrm{MFG}}_{x_0}(\alpha^\gamma, \bar{\nu}^0) = \mathbb{E}\int_0^T \left[\frac{A}{2}|S^\gamma_t|^2 + L\, S^\gamma_t + \frac{C}{2}|\alpha^\gamma_t|^2 + \frac{K^\gamma}{2}|Q^\gamma_t - \alpha^\gamma_t|^2 \right.$$
$$\left. -\; P^{\bar{\nu}^0}_t\,(Q^\gamma_t - \alpha^\gamma_t)\right] dt + \mathbb{E}[g(S^\gamma_T)]$$

where $P^{\bar{\nu}^0}$ is the process defined by

$$P^{\bar{\nu}^0}_t = p\left(-Q^r_t - \sum_{\gamma' \in \Gamma} \pi^{\gamma'}\left(\mathbb{E}[Q^{\gamma'}_t|\mathcal{F}^0_t] - \bar{\nu}^{\gamma',0}_t\right)\right)$$

Definition - Mean field Nash equilibrium

Let $x_0 = (s_0, q_0)$ be a random vector independent from $\mathbb{F}^0$. We say that $\alpha^\star = \{\alpha^{\gamma,\star}, 1 \le \gamma \le \Gamma\}$ is a mean field Nash equilibrium if, for each γ, $\alpha^{\gamma,\star}$ minimizes the function $\alpha^\gamma \mapsto J^{\gamma,\mathrm{MFG}}_{x_0}(\alpha^\gamma, \{\mathbb{E}[\alpha^\star_t|\mathcal{F}^0_t]\})$.

Proposition (Characterization of Mean field Nash equilibria)

Let $\bar{\nu}^0$ be a given $\mathbb{F}^0$-adapted $\mathbb{R}^\Gamma$-valued process, and $x_0 = (s_0, q_0)$ be a random vector which is independent form $\mathbb{F}^0$. Then there exists a unique control $\alpha^\star = (\alpha^{1,\star}, \cdots, \alpha^{\Gamma,\star}) = \alpha^\star(\bar{\nu}^0, x_0)$ such that: for each γ, $\alpha^{\gamma,\star}$ minimizes the function $\alpha^\gamma \mapsto J_{x_0}^{\gamma,\mathrm{MFG}}(\alpha^\gamma, \bar{\nu}^0)$. Moreover, if $(S^{\gamma,\star}, Q^\gamma)$ is the state process corresponding to the initial data condition x_0^γ, to the control $\alpha^{\gamma,\star}$, and to the dynamic 3-2, then there exists a unique adapted solution $(Y^{\gamma,\star}, Z^{0,\gamma,\star}, Z^{\gamma,\star})$ of the BDSE

$$\begin{cases} dY_t^{\gamma,\star} &= -\left(AS_t^{\gamma,\star} + L\right) dt + Z_t^{0,\gamma,\star} dB_t^0 + Z_t^{\gamma,\star} dB_t^\gamma \\ Y_T^{\gamma,\star} &= \partial_s g(S_T^{\gamma,\star}) \end{cases} \tag{4}$$

satisfying the coupling condition

$$Y_t^{\gamma,\star} + p\left(-Q_t^r - \sum_{\gamma'} \pi^{\gamma'}(\bar{Q}_t^{\gamma',0} - \bar{\nu}_t^{\gamma',0})\right) - K^\gamma Q_t^\gamma + (C + K^\gamma)\alpha_t^{\gamma,\star} = 0 \tag{5}$$

Conversely, assume that there exists $(\alpha^{\gamma,\star}, S^{\gamma,\star}, Y^{\gamma,\star}, Z^{0,\gamma,\star}, Z^{\gamma,\star})$ which satisfy the coupling condition 5 as well as the FBSDE 3-2-4, then $\alpha^{\gamma,\star}$ is the optimal control minimizing $J_{x_0}^{\gamma,\mathrm{MFG}}(\alpha^\gamma, \bar{\nu}^0)$ and $S^{\gamma,\star}$ is the optimal trajectory. If in addition we have

$$\mathbb{E}\left[\alpha_t^{\gamma,\star} | \mathcal{F}_t^0\right] = \bar{\nu}_t^{\gamma,0}, \ \ \forall \gamma = 1, \cdots, \Gamma$$

then $\alpha^\star$ is a mean field nash equilibrium.

Let $x_0 = (s_0, q_0) = \{x_0^{\gamma'} = (s_0^{\gamma'}, q_0^{\gamma'}), 1 \leq \gamma' \leq \Gamma\}$ be a random vector independent from $\mathbb{F}^0$ and $J_{x_0}^{\mathrm{MFC}}$ be the cost function defined for $\alpha = (\alpha^1, \cdots, \alpha^\Gamma) \in^\Gamma$ by

$$J_{x_0}^{\mathrm{MFC}}(\alpha) = \mathbb{E}\left[\int_0^T \frac{K^\gamma}{2}\left|Q_t^r\right|^2 - P_t^{\bar{\alpha}^0} Q_t^r dt\right]$$

$$+ \sum_\gamma \pi^\gamma \mathbb{E}\left[\int_0^T \left[\tfrac{A}{2}|S_t^\gamma|^2 + LS_t^\gamma + \tfrac{C}{2}|\alpha_t^\gamma|^2 + \tfrac{K^\gamma}{2}|Q_t^\gamma - \alpha_t^\gamma|^2 \right.\right.$$
$$\left.\left. - P_t^{\bar{\alpha}^0}(Q_t^\gamma - \alpha_t^\gamma)\right] dt\right] + \sum_\gamma \pi^\gamma \mathbb{E}\left[g(S_T^\gamma)\right]$$

where S^γ is defined by 3, Q^γ by 2, and

$$P_t^{\bar{\alpha}^0} := p\left(-Q_t^r - \sum_\gamma \pi^\gamma \left(\mathbb{E}[Q_t^\gamma|\mathcal{F}_t^0] - \mathbb{E}[\alpha_t^\gamma|\mathcal{F}_t^0]\right)\right)$$

Definition (Optimal coordinated plan)

We say that $\hat{\alpha} = (\hat{\alpha}^1, \cdots, \hat{\alpha}^\Gamma) \in \mathcal{A}^\Gamma$ is an optimal coordinated plan if: $\hat{\alpha} = \mathrm{argmin}_{\alpha \in \mathcal{A}^\Gamma} J_{x_0}^{\mathrm{MFC}}(\alpha)$.

Proposition (Characterization of the mean field type control)

There exists a unique optimal control $\hat{\alpha} = (\hat{\alpha}^1, \cdots, \hat{\alpha}^\Gamma)$ *which minimizes the functional* $J^{\mathrm{MFC}}_{x_0}(\alpha)$*. Moreover, if* $\hat{S} = (\hat{S}^{,} \cdots, \hat{S}^\Gamma)$ *is the corresponding controlled trajectory, then there exists a unique adapted solution* $(\hat{Y}, \hat{Z}, \hat{Z}^0)$ *of the BDSE*

$$\begin{cases} d\hat{Y}_t &= -\left(A\hat{S}_t + L_\Gamma\right) dt + \hat{Z}^0_t dB^0_t + \hat{Z}_t dB_t \\ \hat{Y}_T &= \left(\partial_s g(\hat{S}^1_T), \cdots, \partial_s g(\hat{S}^\Gamma_T)\right)^T \end{cases} \tag{6}$$

satisfying the coupling condition: for all $\gamma = 1, \cdots, \Gamma$

$$0 = \hat{Y}^\gamma_t + (C + K^\gamma)\,\hat{\alpha}^\gamma_t + \left(P^{\bar{\alpha}^0}_t - K^\gamma Q^\gamma_t\right)$$

$$\text{-}\, p'(-Q^r_t - \Pi_\Gamma \cdot (\bar{Q}^0_t - \bar{\alpha}^0_t))\,(Q^r_t + \Pi_\Gamma \cdot (\bar{Q}^0_t - \bar{\alpha}^0_t)) \tag{7}$$

with $\bar{\alpha}^0_t = \mathbb{E}[\hat{\alpha}_t | \mathcal{F}^0_t]$.

Conversely, suppose $(\hat{S}, \hat{\alpha}, \hat{Y}, \hat{Z}^0, \hat{Z})$ *is an adapted solution to the forward backward system 3-6, with the coupling condition 7, then* $\hat{\alpha}$ *is the optimal control minimizing* $J^{\mathrm{MFC}}_{x_0}(\alpha)$ *and* $\hat{S}$ *is the optimal trajectory.*

Proposition

Consider the solution $\hat{\alpha}$ of MFC problem with a pricing rule p. Then $\hat{\alpha}$ is a mean field nash equilibrium for the MFG problem with pricing rule

$$p^{\mathrm{MFG}}(x) = p(x) + xp'(x) \ .$$

Proposition

Let $\alpha^{i,\star}$ is a mean field Nash equilibrium for $J^{\mathrm{MFG}}_{x_0^i}$. Then for each $\epsilon > 0$ there exists N_ϵ and η such that: if $N \geq N_\epsilon$ and $\eta \leq \eta_\epsilon$, then $\alpha^\star := (\alpha^{1,\star}, \cdots, \alpha^{N,\star})$ is an ϵ-Nash equilibrium for the N-players game.

Proof: from Graber

Explicit solutions: by assuming that the pricing rule is linear $p: \ x \mapsto p_0 + p_1 x$ and that the terminal cost is quadratic $g: \ s \mapsto \frac{B_2}{2}s^2 + B_1 s + B_0$ we have explicit solution for the MFC and therefore for the MFG.

Numerical examples - some findings

- Time horizon T is typically 1 day or two.
- We consider first one prosumer region whose seasonnality is half the seasonality of the rest of the world.
- Ornstein-Uhlenbeck diffusions for consumption processes.

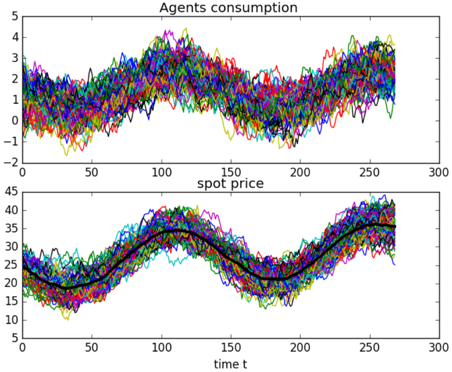


Figure: Agent's consumption (upper figure) and corresponding spot price (low figure) with average prices (wide black line) for several simulations, $T = 1$ day, $a = 1$, $\sigma = 0.8$ and $\sigma^0 = 0.3$.

Numerical examples - some findings

Batteries are used by prosumers both for arbitraging peak/off-peak spot prices and to limit their demand charge part of their electricity bill

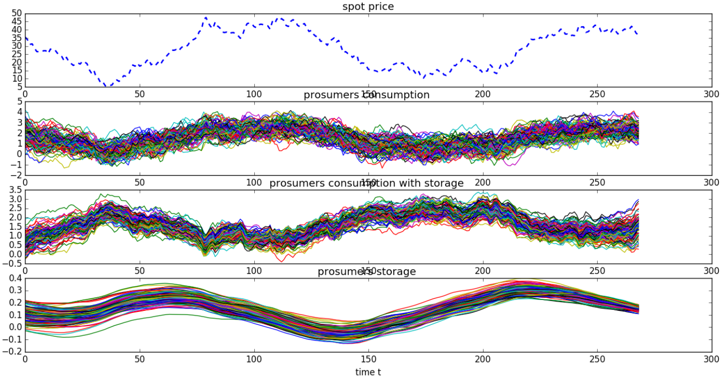


Figure: One simulation of spot price (upper graph), prosumers' consumption Q^i (middle graph), prosumer's net consumption $Q^i - \alpha^i$ (lower middle graph) and prosumer' storage level (lower graph) for every prosumers when **prosumers have no impact on price**.

Numerical examples - some findings

Batteries are used by prosumers both for arbitraging peak/off-peak spot prices and to limit their demand charge part of their electricity bill.

When **no impact on price**, batteries enable

- 27% average reduction of instantaneous power consumption
- 13% bill reduction
- 7% total cost reduction when including storage costs

	electricity bill	reduction implied by battery
volumetric charge	76%	21%
demand charge	24%	8%

prosumers - battery owners

Numerical examples - some findings

Batteries are used by prosumers both for arbitraging peak/off-peak spot prices and to limit their demand charge part of their electricity bill

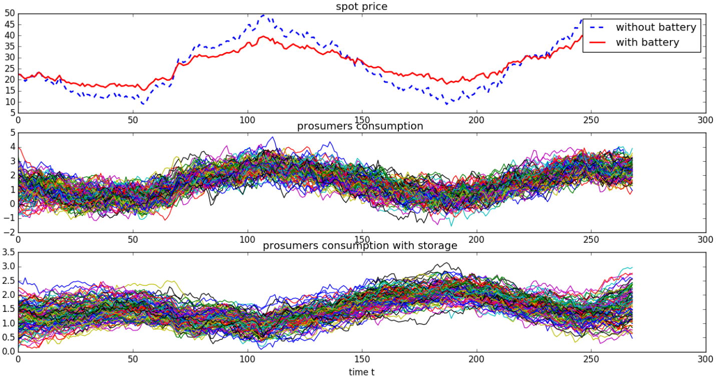


Figure: One simulation of spot price (upper graph), prosumers' consumption Q^i (middle graph), prosumer's net consumption $Q^i - \alpha^i$ (lower middle graph) and prosumer' storage level (lower graph) for every prosumers when **prosumers have equal impact on price as rest of the world**.

Numerical examples - some findings

Batteries are used by prosumers both for arbitraging peak/off-peak spot prices and to limit their demand charge part of their electricity bill.

When **prosumers impact price as rest of the world**, batteries enable

- 30% average reduction of instantaneous power consumption
- 5% bill reduction for rest of the world
- autosufficient prosumers disconnect from the grid if peak/off-peak spread is too little or if high demand charge component
- optimal battery capacity is lower compared to no-impact on price case

	electricity bill	reduction implied by battery
volumetric charge	76%	13%
demand charge	24%	16%

prosumers - battery owners

- Higher volatility prosumers install higher capacity batteries and have a larger reduction of their bill

Numerical examples - some findings

Decentralised management (MFG) leads to having less batteries in the system than what a central planner would do (MFC)

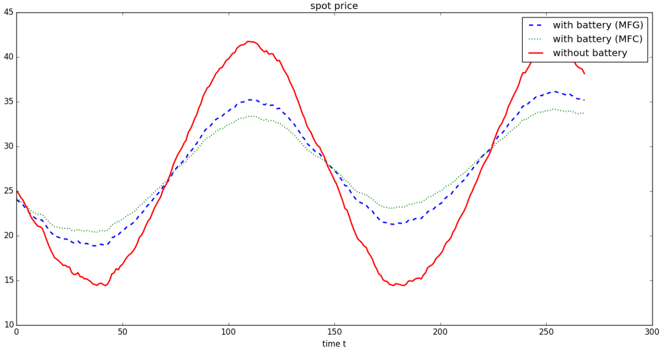


Figure: Average spot price over without battery in the system (straigth line), with decentralized batteries (dashed line) and with batteries optimized by a central planner (dotted line)

Conclusion

Main results:

- We propose in this model a stylized quantitative model for a power grid with distributed local energy generation and storage
- Extended MFG approach provides an analytically and numerically tractable setting to analyze the model
- With quadratic cost structure and linear pricing rule, we provide explicit solutions and existence + unicity results for the equilibrium
- Extended MFG can be linked to suitable Mean Field Type Control (MFC) problem (central planner point of view)

Future research: more realistic numerical implementation, apply the model to consumer's flexibility...