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Wind Speed and Wind Power Forecasting Considering Asymmetric Loss

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Need for models which predict wind speed and power?

Motivation

$\rightsquigarrow$ Is there any need for wind speed and/or power prediction? Wind speed $\neq$ Wind power? The relationship is non-linear!

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The theoretical wind power function is given by

$$\bar{P}_t = \frac{1}{2} A \rho F W_t^3, \qquad (1)$$

1. wind speed W_t in (m/s)
2. the swept area A (m^2) of the wind turbine
3. the efficiency of the turbine F
4. the air density ρ (kg/m^3)

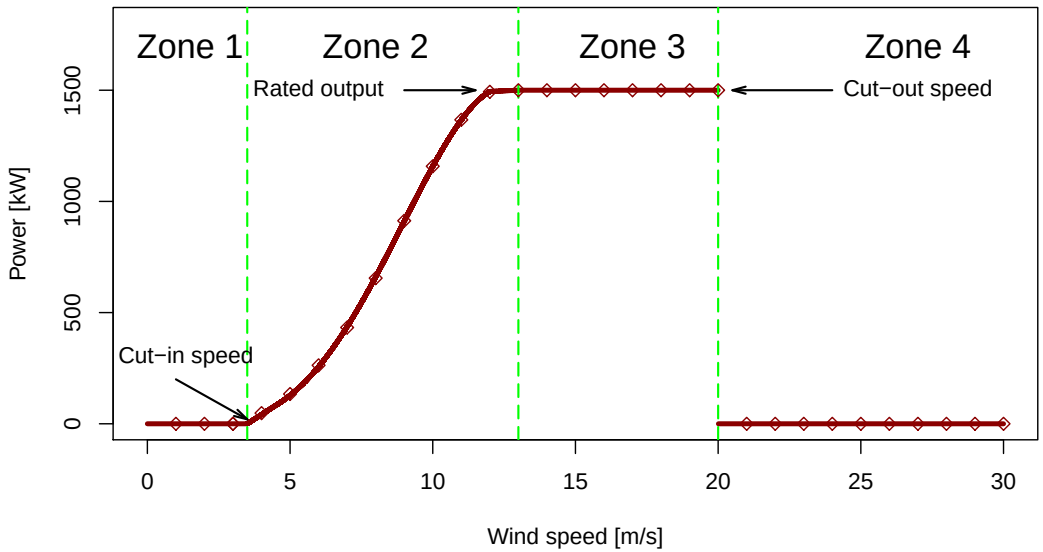

Figure: Power production function of a Fuhrländer MD 77 in kW

Classification of different prediction intervals

Forecasting horizon	Time frame	Usage
Very short-term	$\leq 30min$	• electricity market clearing • regulatory actions
Short-term	$30min < x \leq 6h$	• economic loading, dispatch planning • hint for increment and decrement decisions • decide whether generator is online or offline
Medium-term	$6h < x \leq 24h$	• operational security in day-ahead electricity markets • once again generator online/offline decisions • unit commitment decisions at spot and OTC markets

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Research questions

- necessary to predict wind speed and power
- extend the forecasting performance for wind speed and power for different time spans,
- determine explanatory variables (covariates), e.g. diurnal and annual pattern, variation of air pressure and wind direction
- incorporate persistence

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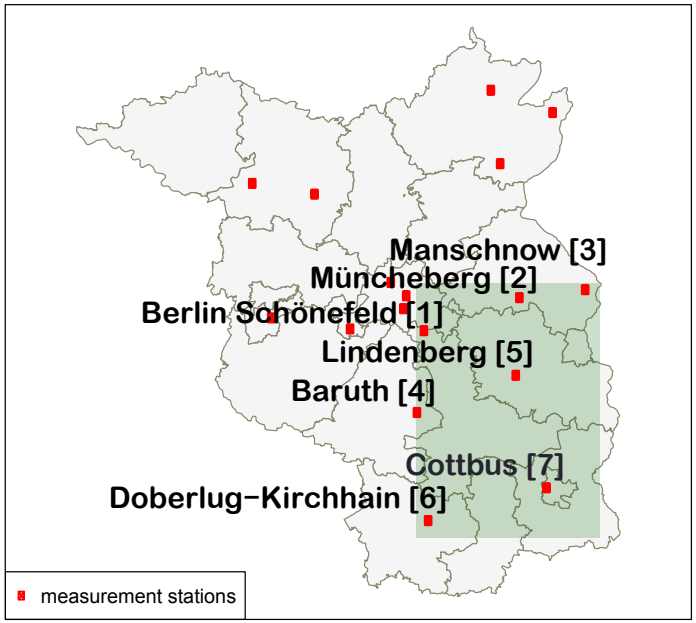
Description of the Wind Speed Data

Wind speed data set (10min) provided by the "Deutscher Wetter Dienst" (DWD) and collected with an anemometer

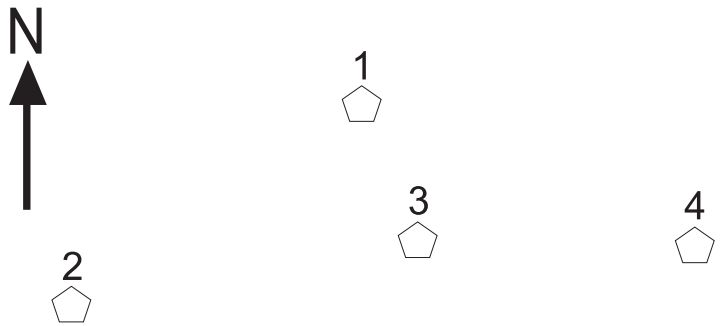
First Application - Multivariate Space-Time Wind Speed Prediction **the data set consists of about** $160,000$ **observations for station**

- first up to 107.000 observations for model identification *(January 2009 - June 2011)*,
- approximately 53.000 observations are used for out-of-sample forecasts *(July 2011 - end of December 2011)*,



Second Application - Multivariate Space-Time Wind and Power Prediction $\Rightarrow$ **for each turbine we observe** $T = 105\,984$ **observations**

- spatial dimension $m \in \{1, \ldots, M\}$ and 4 turbines $M = 4$,
- time dimension $t \in \{1, \ldots, T\}$,
- up to 82 300 observations for model identification *(1. November 2010 - 5. November 2012)*,
- approximately 25 000 observations are used for out-of-sample forecasts *(1. July 2012 - 1. November 2012)*,



	Wind speed	Wind power	Azimuth	Temperature
Minimum	0.40	-19.00	2.00	-17.00
25% Quantile	2.53	0.37	84.10	4.00
Median	5.06	223.63	189.16	10.00
Mean	6.04	352.26	173.22	9.81
75% Quantile	6.60	318.00	245.00	17.00
Maximum	19.50	1542.00	357.00	38.00
Standard deviation	5.63	497.42	108.60	15.48

Brief Summary of Wind Speed Characteristics

- Nonnegative and nonnormal (Probability density function),
- Huge temporal correlation,
- Diurnal, seasonal and annual changes,
- Changes rapidly and with high frequency,
- Spatially correlated.

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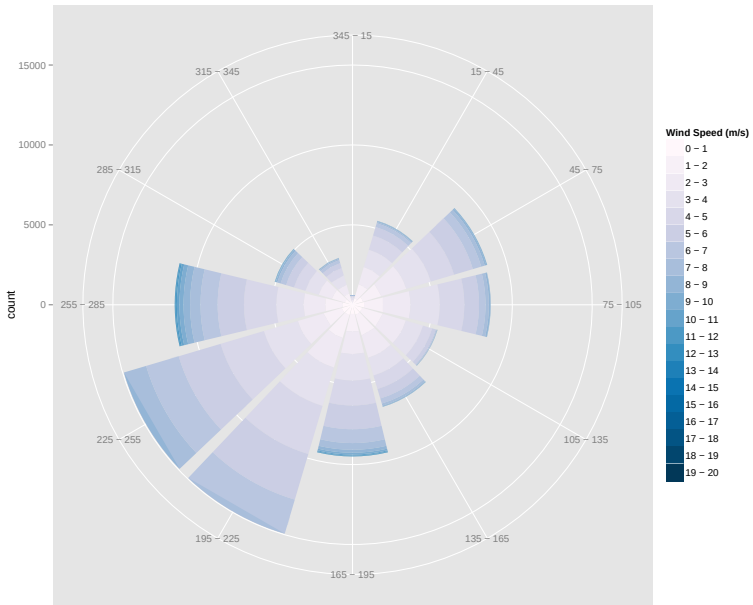

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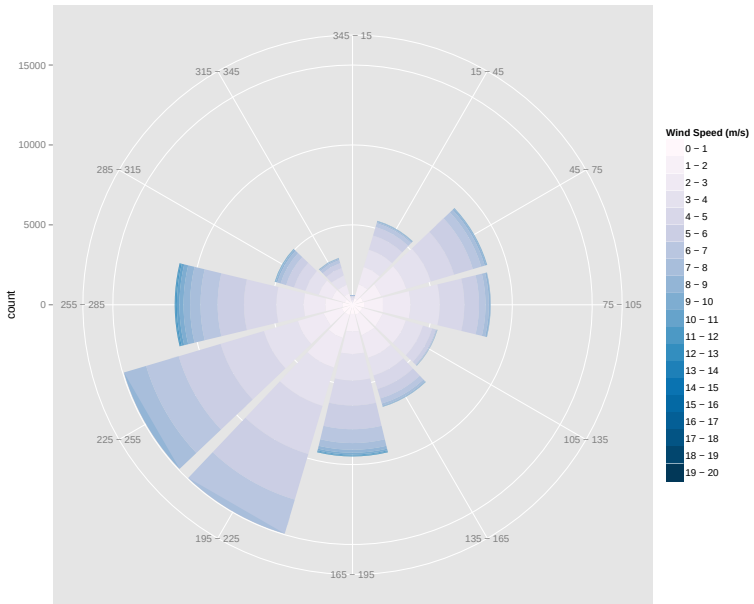
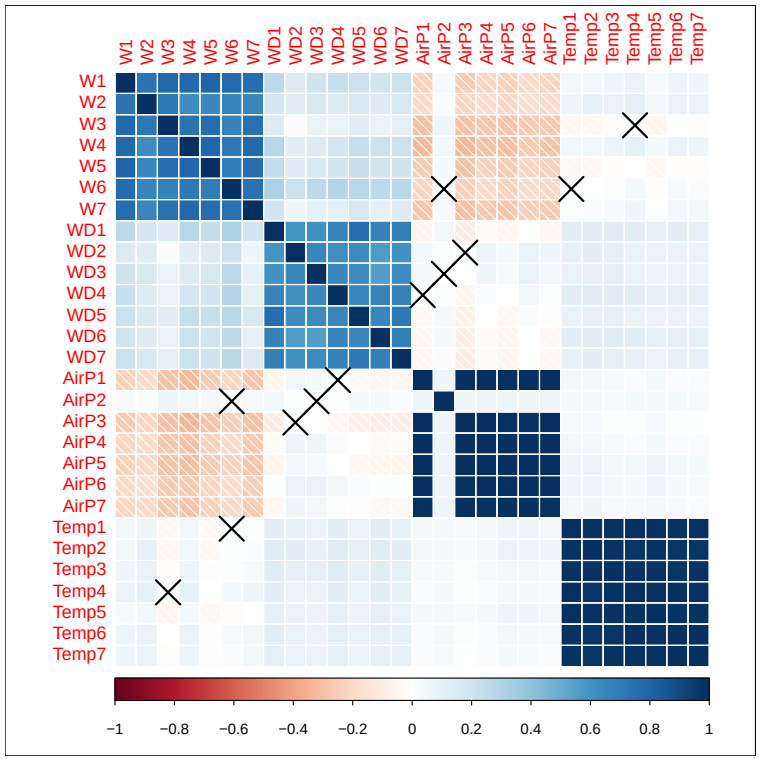




Modelling Equation

For stations $m = 1, \ldots, M$ and time point $t = 1, \ldots, T$, the dependent vector consists of observations for 5 variables, i.e. $\boldsymbol{Y}_t \in \mathbb{R}^{5 \cdot M}$, which is

$$\begin{aligned} \boldsymbol{Y_t} = \quad & \Big(W_{1,t}, \ \ldots, \ W_{M,t}, \quad \sin(az_{1,t}), \ \ldots, \ \sin(az_{M,t}), \\ & \cos(az_{1,t}), \ \ldots, \ \cos(az_{M,t}), \quad C_{1,t}, \ \ldots, \ C_{M,t} \Big)'. \end{aligned} \tag{2}$$

- $az_{M,t}$ is the azimuth of station $m \in \{1, ..., M\}$,
- $C_{M,t}$ for all stations includes the temperature in $^\circ C$,
- decompose our wind direction information into an east-west component, given by $\sin(az_{m,t})$,
- a north-south component, given by $\cos(az_{m,t})$.

Modelling Equation (cont.)

$$\boldsymbol{Y}_t = \beta_t + \sum_{j=1}^{J} \Theta_{t,j} \boldsymbol{Y}_{t-j} + \epsilon_t \,, \tag{3}$$

$$\beta_t = \beta_0 + \sum_{k_1=2}^{K_1} \beta_{k_1,1} f_{k_1}^{s_1}(t) + \sum_{k_2=2}^{K_2} \beta_{1,k_2} f_{k_2}^{s_2}(t) + \sum_{k_1=2}^{K_1} \sum_{k_2=2}^{K_2} \beta_{k_1,k_2} f_{k_1}^{s_1}(t) f_{k_2}^{s_2}(t) \,, \tag{4}$$

$$\Theta_{t,j} = \Theta_{0,j} + \sum_{k_1=2}^{K_1} \Theta_{k_1,1,j} f_{k_1}^{s_1}(t) + \sum_{k_2=2}^{K_2} \Theta_{1,k_2,j} f_{k_2}^{s_2}(t) + \sum_{k_1=2}^{K_1} \sum_{k_2=2}^{K_2} \Theta_{k_1,k_2,j} f_{k_1}^{s_1}(t) f_{k_2}^{s_2}(t) \,, \tag{5}$$

$$\epsilon_t = \sigma_t \eta_t \,. \tag{6}$$

- Azimuth 0° or 360° is the north and $f_{k_1}^{s_2}(t)$ and $f_{k_1}^{s_2}(t)$ are periodic B-spline functions,
- $(\boldsymbol{\eta}_t)_{t \in \mathrm{Z}}$ is i.i.d. $\mathrm{E}(\boldsymbol{\eta}_t) = 0$ and $\mathrm{Var}(\boldsymbol{\eta}_t) = \mathbf{1}$,
- $\boldsymbol{\sigma}_t$ can follow an TARCH model (see for example Glosten et al. (1993)).

Variance Model

$$\sigma_t = \alpha_0 + \sum_{k_1=2}^{K_1} \alpha_{k_1} f_{k_1}^{s_1}(t) + \sum_{h=1}^{P} \gamma_{t,h} \boldsymbol{I}_t^+ \epsilon_{t-h} + \sum_{l=1}^{Q} \psi_{t,l} \boldsymbol{I}_t^- \epsilon_{t-l}\,, \tag{7}$$

$$\gamma_{t,h} = \gamma_{0,h} + \sum_{k_1=2}^{K_1} \gamma_{k_1,h} f_{k_1}^{s_1}(t)\,, \tag{8}$$

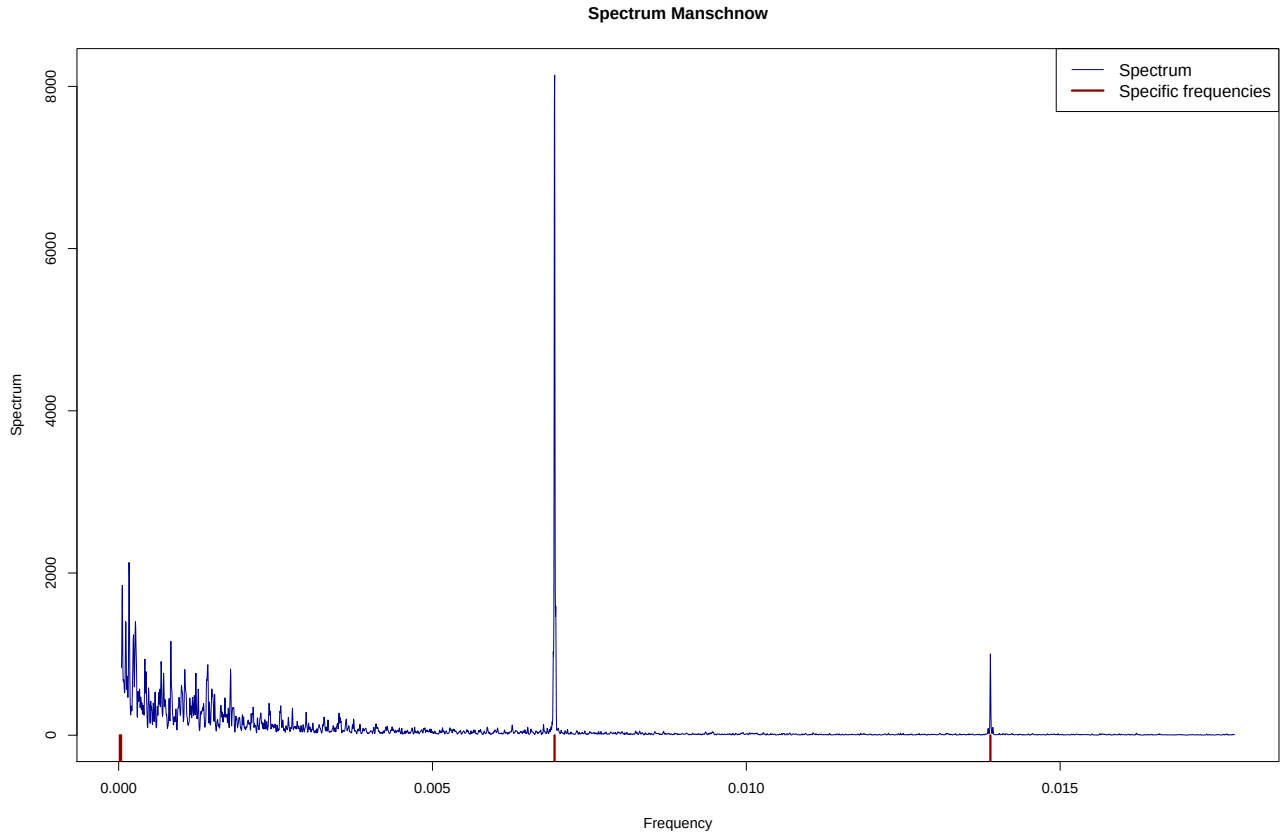
$$\psi_{t,l} = \psi_{0,l} + \sum_{k_1=2}^{K_1} \psi_{k_1,l} f_{k_1}^{s_1}(t)\,. \tag{9}$$

$$I_{n,m,t-1}^+ = \begin{cases} 1, & \epsilon_{n,m,t-1} > 0 \\ 0, & \epsilon_{n,m,t-1} \leq 0 \end{cases} \quad \text{sowie} \quad I_{n,m,t-1}^- = \begin{cases} 1, & \epsilon_{n,m,t-1} \leq 0 \\ 0, & \epsilon_{n,m,t-1} > 0 \end{cases}\,. \tag{10}$$

where $N \in \{1, \ldots, 5\}$ represents the Nth regressor.

- $\boldsymbol{I}_t^+ = \left(I_{1,1,t}^+, I_{1,2,t}^+, \ldots, I_{1,M,t}^+, I_{2,1,t}^+, \ldots, I_{N,M,t}^+\right)'$
- $\boldsymbol{I}_t^- = \left(I_{1,1,t}^-, I_{1,2,t}^-, \ldots, I_{1,M,t}^-, I_{2,1,t}^-, \ldots, I_{N,M,t}^-\right)'$

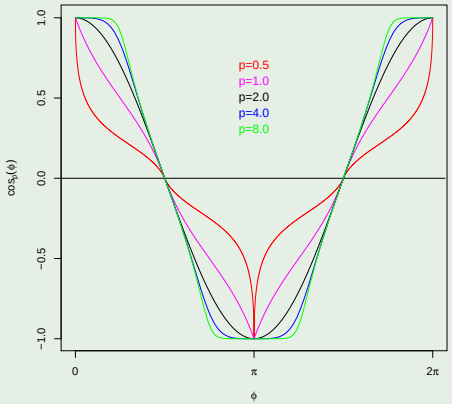
Periodic Functions and Modelling Perspectives

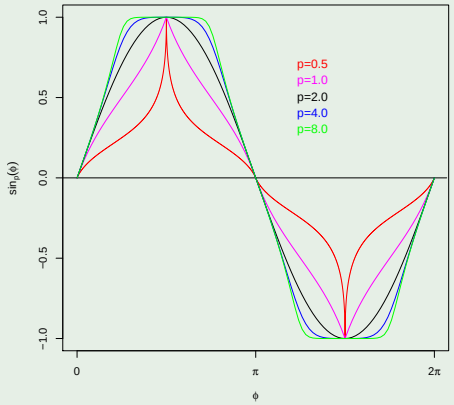


Spectrum , where $\omega = 1/S$
$\omega_1 = 0.006944$, $\omega_1 = 0.01388$
$\omega_3 = 0.00001901$, $\omega_4 = 0.000038$

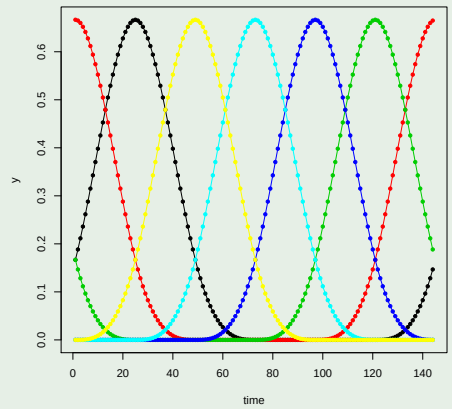
- Diurnal and seasonal changes,

Generalised trigonometrical functions





B-Spline functions



The fundamental basics of the spline functions are defined by De Boor (1978) and Eilers and Marx (1996)

- define the set of equidistant knots
- here annual and diurnal periods are used
- the figure provides diurnal, cubic and periodic B-splines

Considering the parameter vector $\theta_{\boldsymbol{N,m}}$

Lasso estimator

$$\widehat{\boldsymbol{\theta}}_{N,m} = \underset{\boldsymbol{\theta}_{N,m} \in \mathbb{R}^{p_j}}{\arg\min} \sum_{t=1}^{T} (Y_{N,m,t} - \omega_t \boldsymbol{X}_{N,m,t} \boldsymbol{\theta}_{N,m,t})^2 + \lambda_{N,m,T} \sum_{j=1}^{p_j} |\theta_j|, \tag{11}$$

- ω_t is the weight vector $\omega = (\omega_1, ..., \omega_T)$
- p_j number of elements of the parameter vector $\boldsymbol{\theta}_{N,m}$
- tuning parameter is $\lambda_{N,m,T} \geq 0$

Elastic net

The second estimation method is the elastic net which is given by

$$\begin{aligned} \widehat{\boldsymbol{\theta}}_{N,m} = \underset{\boldsymbol{\theta}_{N,m} \in \mathbb{R}^{p_j}}{\arg\min} \textstyle\sum_{t=1}^{T} (Y_{N,m,t} - \omega_t \boldsymbol{X}_{N,m,t} \boldsymbol{\theta}_{N,m,t})^2 \quad & + \lambda_{N,m,T} \alpha \textstyle\sum_{j=1}^{p_j} |\theta_j| \\ & + \tfrac{1}{2} \lambda_{N,m,T} (1-\alpha) \textstyle\sum_{j=1}^{p_j} \theta_j^2, \end{aligned} \tag{12}$$

- $\alpha \in [0, 1]$ additional tuning parameter ($\alpha = 1$, we obtain the lasso case, if $\alpha = 0$, we obtain the ridge regression and if $\alpha \in (0, 1)$ represent the elastic net).

The estimation is done in two-steps by re-weighting the mean model

(1) Set an initial $1 \times T$ weight $\omega = (1, \ldots, 1)$ and set the iteration parameter to $\mathcal{K} = 1$

(2) Estimate the mean using either lasso or elastic net with weights ω

(3) Estimate $\boldsymbol{\sigma_t}$ by use of the conditional variance using again the lasso or elastic net and $\{|\widehat{\epsilon_t}|\}$

(4) Subsequently, redefine the vector ω by $\omega = \left(\left(\widehat{\sigma}_1^{-2}\right), ..., \left(\widehat{\sigma}_T^{-2}\right)\right)$ and use the values obtained by (3).

(5) The convergence of $\boldsymbol{\sigma_t}$ is used as abort criterion if $\Delta_{\mathcal{K}} = ||\widehat{\sigma}_{\mathcal{K}-1} - \widehat{\sigma}_{\mathcal{K}}|| < 10^{-3}$.

(6) Stop the algorithm.

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Modifications

$\rightsquigarrow$ advantage of this approach is the fast computing time, and further extensions are possible
- $\rightsquigarrow$ here, the specific lags have to be chosen,
  - $\rightsquigarrow$ otherwise complex models with multivariate Student's t-distribution or copulas

Accuracy of Wind Speed Forecasting Model

Forecasting Wind Speed and Power

Remarks to Forecasting Procedure:
choose $\nu = 1000$ random points in the forecasting frame $\tau^{(i)}$ für $i = 1, \ldots, \nu$

$$RMSE_o = \sqrt{\frac{1}{\nu}\sum_{i=1}^{\nu}\left(Y_{\nu,m,\tau^{(i)}+o} - \widehat{Y}_{\nu,m,\tau^{(i)}+o}\right)^2}, \tag{13}$$

$$MAE_o = \frac{1}{\nu}\sum_{i=1}^{\nu}\left|Y_{\nu,m,\tau^{(i)}+o} - \widehat{Y}_{\nu,m,\tau^{(i)}+o}\right|. \tag{14}$$

Accuracy of First Application

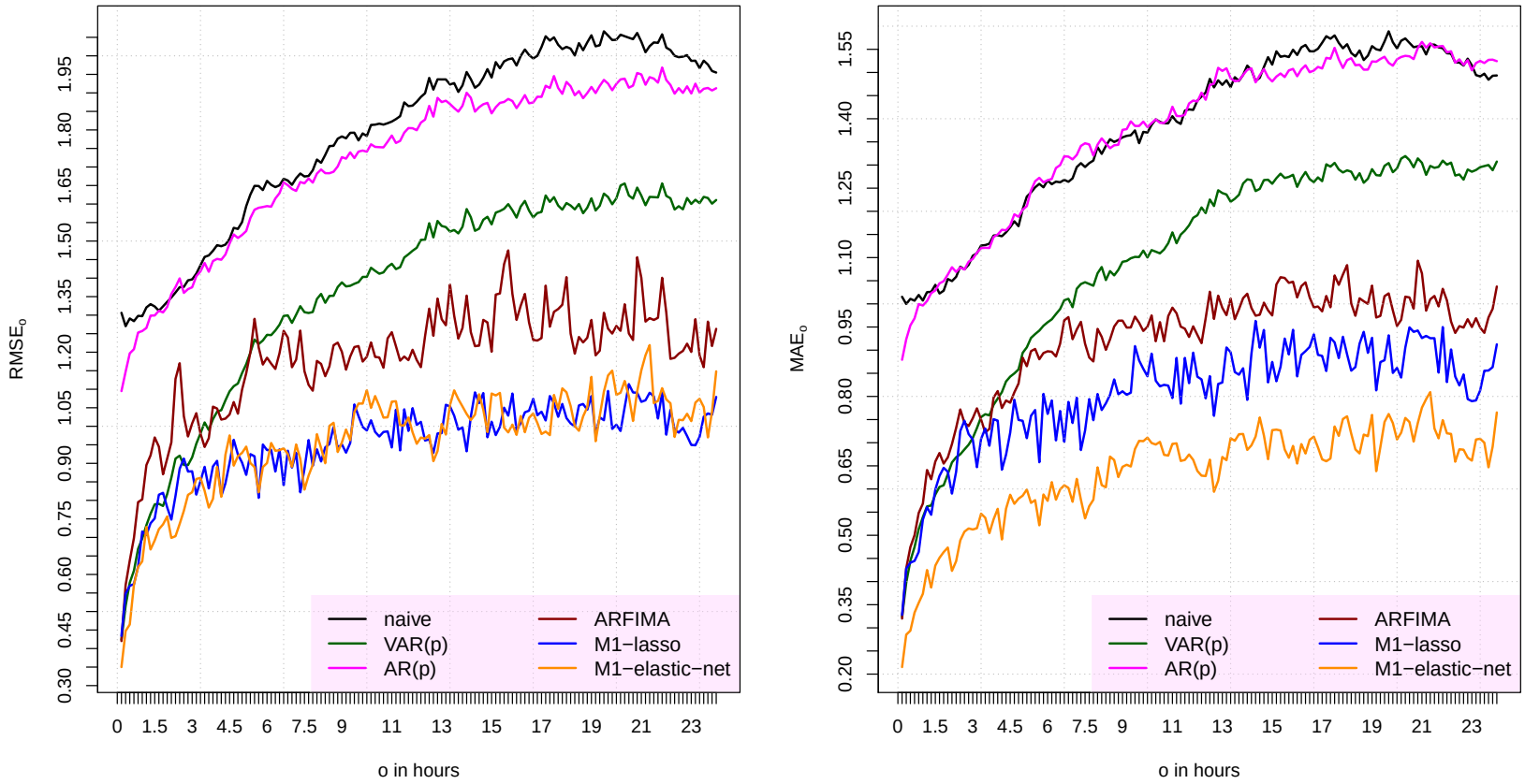

Figure: Root mean squared error for station Müncheberg

Accuracy of First Application (cont. 1/3)

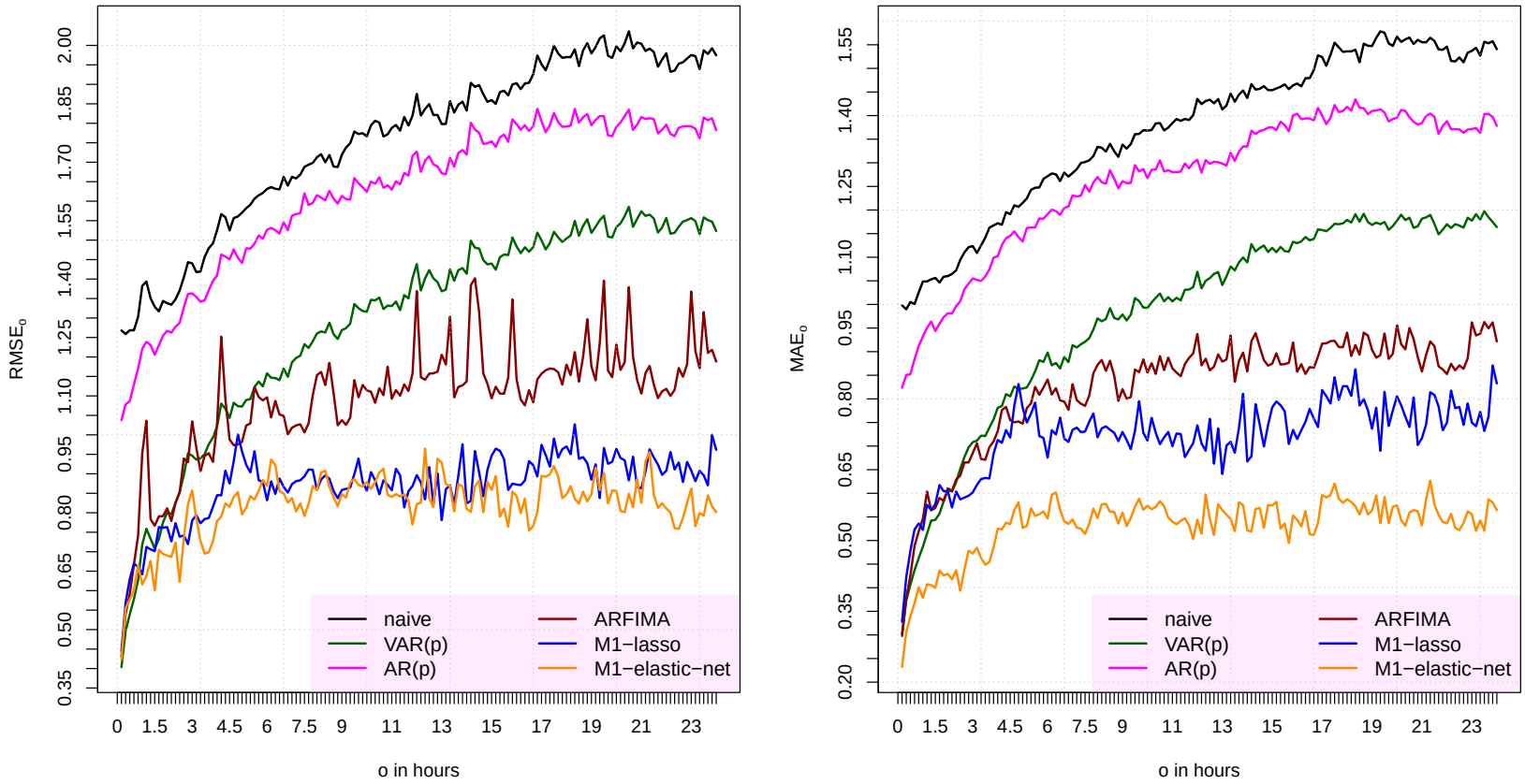

Figure: Root mean squared error for station Lindenberg

Accuracy of Wind Speed Forecasts (cont. 2/3)

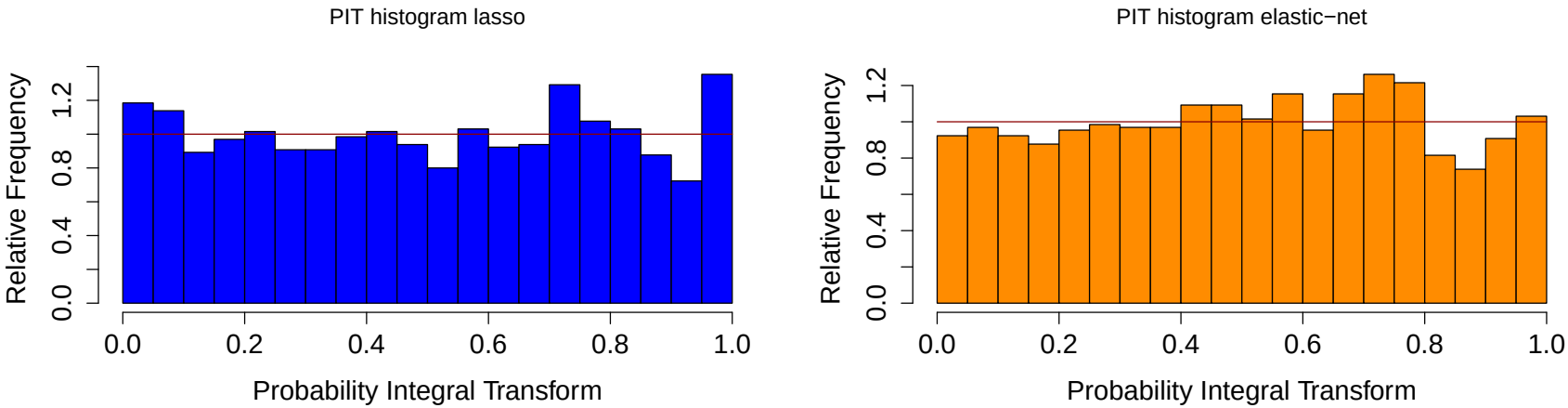

Figure: Probability integral transform histogram of the iteratively reweighted lasso forecasting and elastic net (for 1h and for 4h Müncheberg)

Accuracy of Wind Speed Forecasts (cont. 3/3)

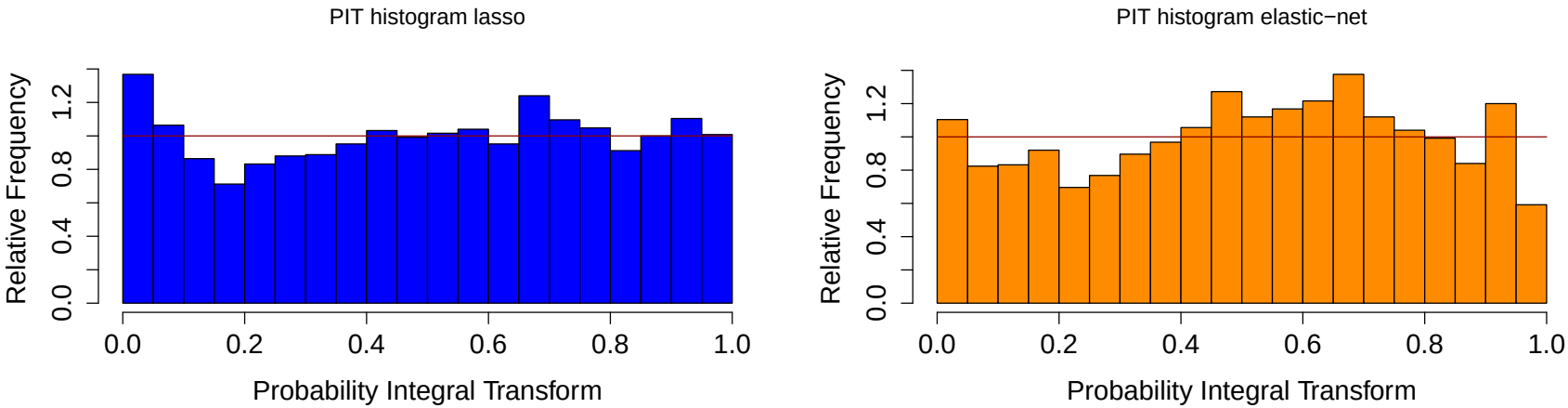

Figure: Probability integral transform histogram of the iteratively reweighted lasso forecasting and elastic net (for 1h and for 4h Lindenberg)

Modelling Equation

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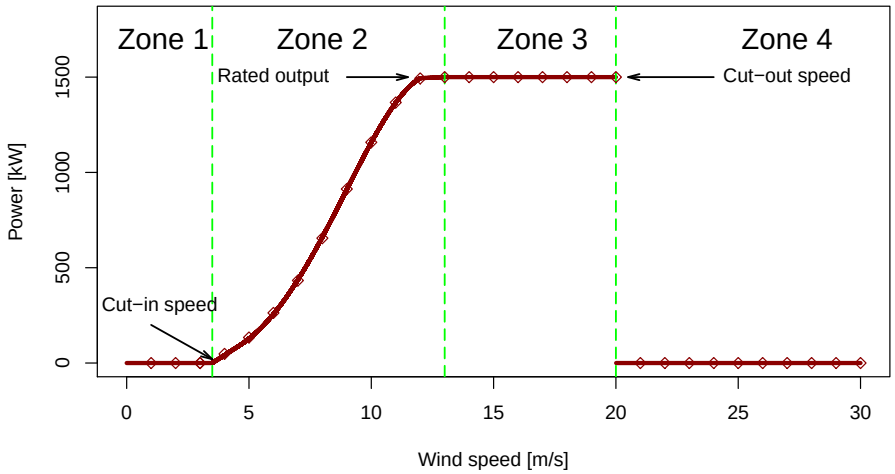
Mapping wind speed in wind power by Fuhrländer wind turbine

$$\tilde{P}_{m,t} = \frac{1}{2} C_m \rho F_m (W_{m,t})^3, \tag{16}$$

$\rightsquigarrow$ measuring the accuracy by RMSE and MAE,

$\rightsquigarrow$ here OLS, lasso and reweighted lasso methods are compared.



Accuracy of Transformed Wind Power Forecasts

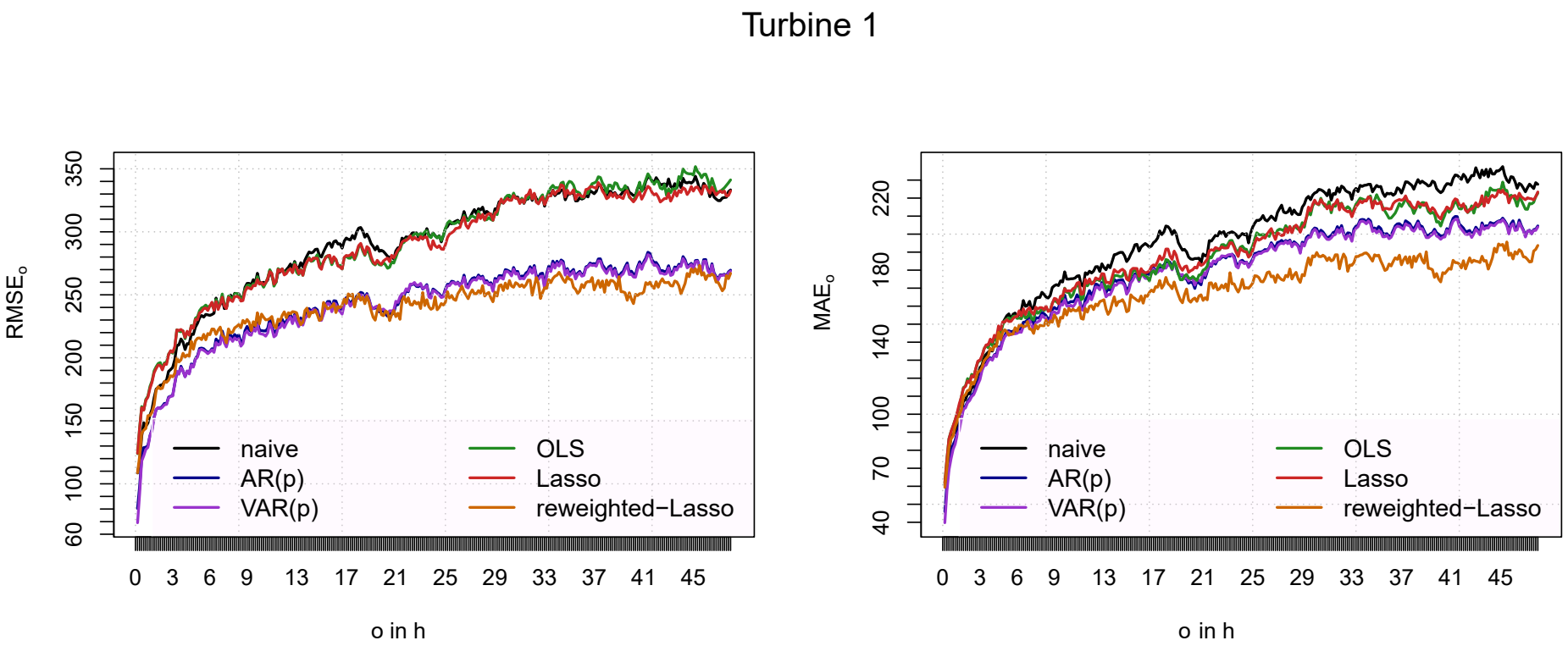
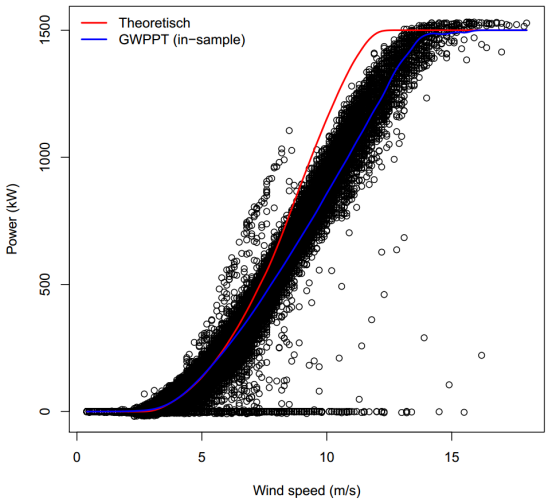


Figure: Root mean squared error and Mean absolute error for each turbine and lag $o \in \{1, \ldots, 288\}$ in hours



$\rightsquigarrow$ usually the RMSE or MAE is taken as loss function, but maybe the electricity market face different rules

- electricity markets as for example NASDAQ, APX, EEX and UKPX need accurate power predictions
- at the electricity spot market, traders place bids on their respective trades
- sellers are obligated to fulfil the contract and to deliver the amount of power
- a time frame of 36h is relevant

+ overestimation: electricity sellers face a loss if sellers deliver less electricity as they have contracted
+ underestimation leads to a smaller amount of power which is sold at the market and producers have to sell their additional amount of power at the market and face a loss due to a lower price
+ underestimation: a system operator has to order too much electricity from the market and have a surplus of electricity
+ system operators have to reduce the generation which is more expensive compared to up-regulation

Asymmetric piecewise linear loss function

A suggested empirical weight $\tau \in [0;1]$ is given by $\tau = .73$ for the magnitude of asymmetry.
$\rightsquigarrow$ i.e. underestimation has to be punished stronger
The asymmetric accuracy measure is given by

$$L\left(P_{t+1}, \widehat{P}_{t+1}\right) = \begin{cases} \tau\left(\widehat{P}_{t+1} - P_{t+1}\right), & \widehat{P}_{t+1} \leq P_{t+1} \\ (1-\tau)\left(P_{t+1} - \widehat{P}_{t+1}\right), & \widehat{P}_{t+1} > P_{t+1}. \end{cases} \tag{17}$$

- $\widehat{P}_{t+1}$ is the forecast of wind power

comparison study

- for the benchmark models $\widehat{P}_{t+1}$ is obtained directly
- for the univariate and multivariate model the forecasts are calculated with use of the power function
- the wind speed predictions $\widehat{W}_{t+h}$ for the forecasting horizon h are mapped by power function to obtain $\widehat{\widehat{P}}(\widehat{W}_{t+1})$
- underestimation is penalized stronger than overestimation, i.e. $\tau = .73$.
- in addition different values of $\tau = .6, .73, .9$ are used

Accuracy of Wind Power Forecasts

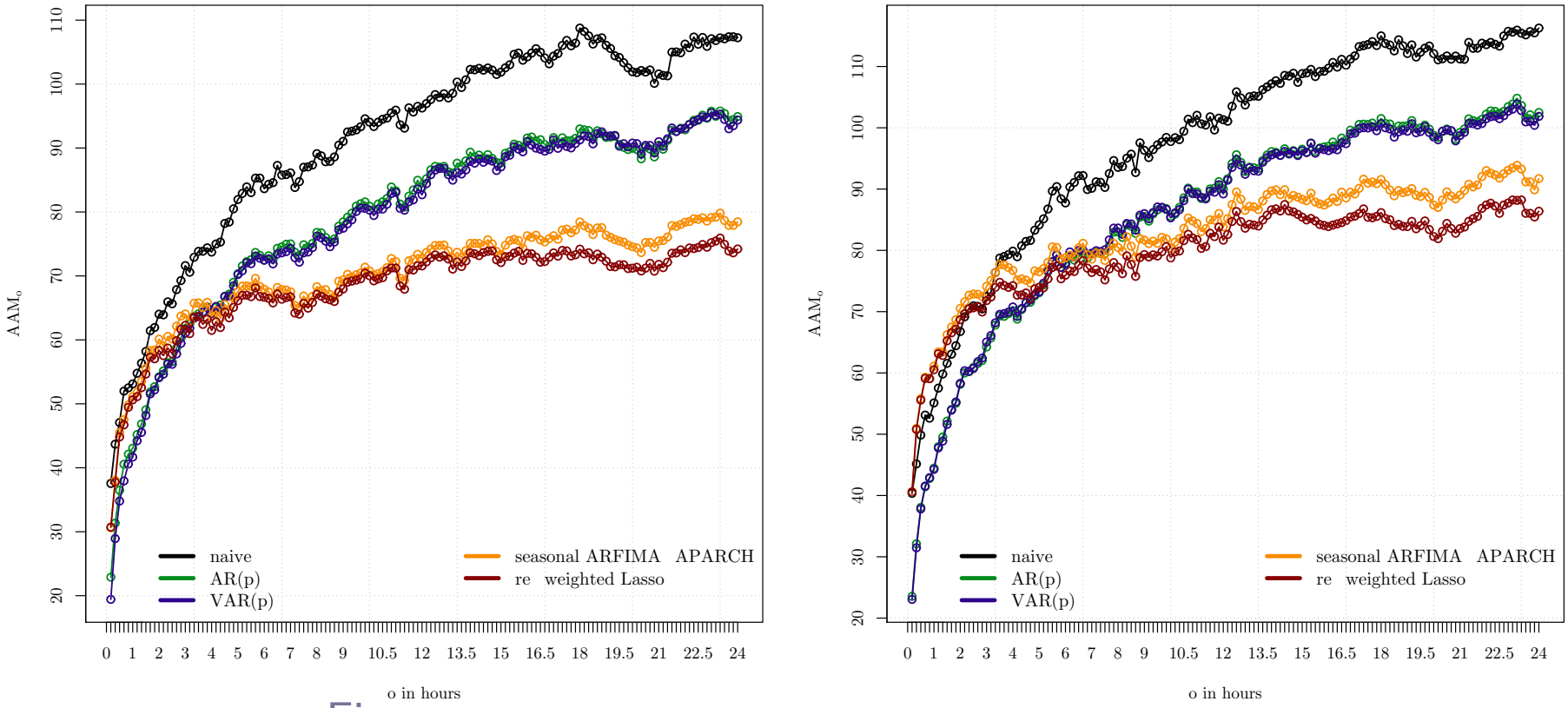


Figure: AAM for 2 turbines $\tau = .6$ and lag $o \in \{1, \ldots, 288\}$ in hours

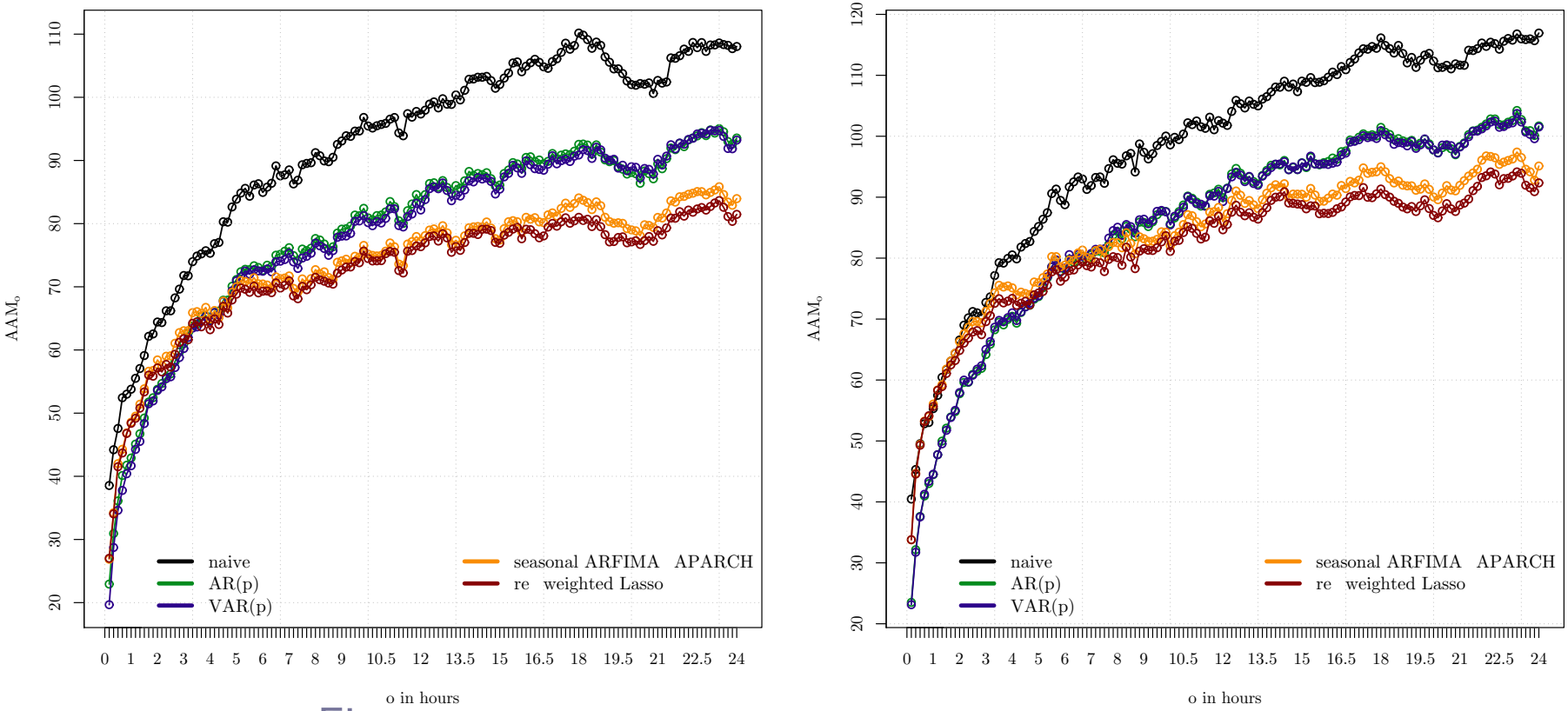


Figure: AAM for 2 turbines $\tau = .73$ and lag $o \in \{1, \ldots, 288\}$ in hours

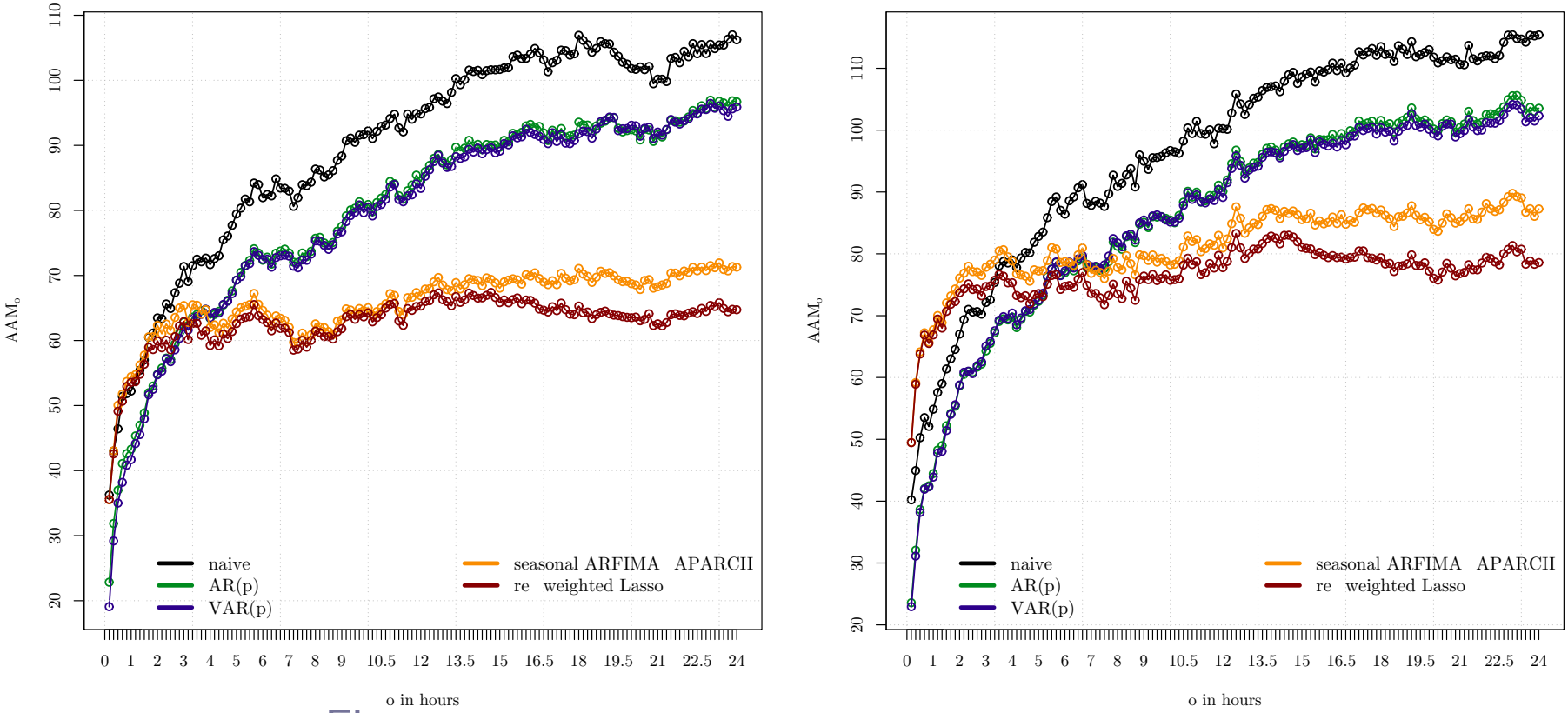


Figure: AAM for 2 turbines $\tau = .9$ and lag $o \in \{1, \ldots, 288\}$ in hours

Conclusion

- importance of informations and covariates,
- multivariate approach (periodic VARX) is an improvement and computationally less intensive
- elastic net improves the lasso method, but is slower
- transformed power forecasts obtained by re-weighted lasso method provide good forecasts
- considering asymmetry underestimation has to be punished more compared to overestimation
- considering the lasso method helps to achieve suitable medium term predictions under asymmetric loss

Outlook: Spatial vs Temporal content

- possible spatio-temporal extension could be provided by functional regression,
- inter-diurnal time-points provide a function of the station, the day and the 10
- hybrid approach with neural network

$$
\begin{aligned}
W(j,t,s) &= \text{single observation,} \\
&\quad s \in \{1,\ldots,M\} - \text{stations,} \\
&\quad t \in \{1,\ldots,N\} - \text{daily period,} \\
&\quad j \in \{1,\ldots,144\} - \text{within a day (10min),} \\
W_{t,s}(j) &= \text{realisation of a function,}
\end{aligned}
$$

So the observations are a vector $\begin{pmatrix} W_{t,s}(1) \\ \vdots \\ W_{t,s}(144) \end{pmatrix}$

Thank you for your attention.

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