Modeling Spot Price Dependence in Australian Electricity Markets with Applications to Risk Management

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1 Introduction
2 The Australian National Electricity Market (NEM)
3 Copulas
4 Empirical Analysis
5 Conclusions
Motivation

- The integration of regional electricity markets recently has gained some interest in the literature.
- Australia operates the longest interconnected electricity grid in the world.
- Main market players—both producers and retailers—usually operate in several regional markets.
- Prices show similar patterns but often also deviate significantly in terms of overall level, volatility.
- Extreme observations such as price spikes seem to happen at the same time in regional markets.
- To our best knowledge no study has investigated the dependence structure between regional electricity markets using copulas.
Studies on integration and dependence between regional electricity markets

- De Vany and Walls (EE, 1999) find cointegration and some evidence for a pattern of nearly uniform prices in different regional U.S. markets.
- Park et al. (EE, 2006) investigate the connection between 11 U.S. spot markets using vector autoregression and find that relationship is dependent on transmission lines and similar institutional arrangements.
- Haldrup and Nielsen (JEcon, 2006) use Markov regime switching models in combination with long memory processes to model the dependence between pairs of regional electricity prices in the NordPool market.
- Dempster et al. (EE, 2008) apply Granger causality tests and cointegration analysis to Californian electricity markets find only a moderate degree of integration.
Studies on integration and dependence between regional electricity markets

- Bollino and Polinori (2008) identify significant contagion effects in regional electricity markets in Italy.
- Zachmann (EE, 2008) examines to which extent European electricity wholesale day-ahead prices converge towards arbitrage freeness.
- Le Pen (2008) uses VAR-BEKK model and finds evidence of return and volatility spillovers between the German, the Dutch, and the British forward electricity market.
- Worthington and Higgs (EE, 2005), Higgs (EE, 2009) employ multivariate GARCH and DCC MGARCH models to investigate the price and volatility spillovers in Australian electricity markets.
The Australian National Electricity Market (NEM) covers the Eastern and Southern interconnected electricity system with 6 price regions.

**Figure:** The Australian National Electricity Market (NEM) covers the Eastern and Southern interconnected electricity system with 6 price regions.
The Australian National Electricity Market (NEM)

- NEM includes six price regions: Queensland (QLD), New South Wales (NSW), Victoria (VIC), South Australia (SA), Tasmania (TAS) and Australian Capital Territory (ACT)
- Prices show similar patterns but sometimes deviate significantly in terms of overall level, volatility, observed price spikes etc.
- Main market players - both producers and retailers - usually operate in several regional markets
- Retailers and customers pay regional prices
- Our study focuses on the four major markets QLD, NSW, VIC and SA
NEM Dependence Structure

**Figure:** Deseasonalized standardized log-prices for NSW and QLD electricity markets at maximal dependence on 31/07/2006(11:30) (left panel) and minimal dependence on 30/04/2007(16:00) (right panel).
Figure: Deseasonalized standardized log-prices for NSW, QLD, SA, TAS and VIC on 20/07/2006 (left panel) and on 07/12/2009 (right panel).
Dependence structure of financial variables

A copula is a function that combines marginal distributions to form a joint multivariate distribution. Concept was initially introduced by Sklar (1959).

- Correlation usually does not appropriately describe the dependence structure between financial assets, see e.g. Cherubini and Luciano (2001), Jondeau and Rockinger (2006), Junker et al (2006)
- Asset returns are more highly correlated during volatile markets and during market downturns (Longin and Solnik, 2001)
- Strength of the copula comes from its feature that it does not have any assumptions on the joint distributions among the financial assets (Dowd, 2004)
Examples of Copulas

**Figure:** Simulated $U(0, 1)$ with $\tau = 0.7$ for Gaussian (upper left), Student $t$ with $\nu = 4$ (upper right), Clayton (lower left) and Gumbel copula (lower left panel)
Considered copulas and tail dependence

1. **Elliptical Copulae**
   - **Gaussian Copula** (no tail dependence)
     \[ C^G_{\Psi}(u_1, \ldots, u_d) = \Phi_{\Psi}\left\{\Phi^{-1}(u_1), \ldots, \Phi^{-1}(u_d)\right\} \]
     where \( \Phi_{\Psi} \) is the \( d \)-dimensional standard normal cdf.
   - **Student-t Copula** (symmetric tail dependence)
     \[ C^t_{\nu, \Psi}(u_1, \ldots, u_d) = t_{\nu, \Psi}\left\{t^{-1}_{\nu}(u_1), \ldots, t^{-1}_{\nu}(u_d)\right\} \]
     where \( t_d(\nu, 0, \Psi) \) is Student-t cdf, \( \Psi \) is the correlation matrix, \( \nu \) df.

2. **Archimedean Copulae, Mixture Copula Models**
   - **Clayton** (lower tail dependence) \( \theta \in (0, \infty) \)
   - **Gumbel** (upper tail dependence) \( \theta \in (1, \infty) \)

3. **Survival Copulae**
   \[ C^*(u_1, u_2) = 1 - u_1 - u_2 + C(1 - u_1, 1 - u_2) \]
   - survival **Clayton** (upper tail dependence)
   - survival **Gumbel** (low tail dependence)
Multivariate normal vs. copula approach

- The conditional distribution of log-prices is multivariate normal: \( X_t \sim N(0, \Sigma_t) \)
- Drawbacks:
  - does not allow to generate tail dependence
  - does not allow heavy tails
- The conditional distribution of risk factors is modeled with Copula \( C \):
  \[
  X_t \sim C\{F_{X_1}(x_1), \ldots, F_{X_d}(x_d), \theta_t\}
  \]
  - \( F_{X_1}, \ldots, F_{X_d} \) are marginal distributions
  - \( \theta_t \) dependence parameter

\[\downarrow\]
- Specify marginal distributions
- Specify dependence structure
The process $\{X_t\}_{t=1}^T$ of logarithmic deseasonalized electricity prices can be modeled as AR(1)-GARCH(1,1):

$$X_t = a_0 + a_1 X_{t-1} + \varepsilon_t, \quad \text{where} \quad \varepsilon_t = \sigma_t u_t$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2$$

and $\alpha_0 > 0, \alpha_1 \geq 0, \beta_1 \geq 0, \alpha_1 + \beta_1 < 1$

- $\varepsilon_t$ are i.i.d. innovations with $E[\varepsilon_t \mid F_t] = 0$ and $Var[\varepsilon_t \mid F_t] = \sigma_t^2$, where $F_t$ is the available information at time $t$

- $\varepsilon_j, j = 1, \ldots, d$ have continuous marginal distributions $F_j$

- Joint distribution function is given by $F_\varepsilon = C_\theta\{F_1, \cdots, F_d\}$

- Estimation is conducted using the Inference for Margins method
Data Set

Data used

- Time series data
  - Daily electricity spot prices from five electricity markets in Australia: NSW, QLD, SA, TAS and VIC

- Sample period covers from January 1, 2006 to January 1, 2010

- To remove the long-term seasonal component we use wavelet smoothing

- Weekly periodicity is removed by applying a moving average technique as discussed in e.g. Brockwell and Davis (2002); Weron (2006)
Figure: The system price for NSW (top left), the log-price together with a long-term seasonal component obtained by wavelet filtering (top right), the log-price after removing long- and short-term seasonal pattern (bottom left), and the returns obtained by differencing the deseasonalized log-price (bottom right).
Correlations structure

<table>
<thead>
<tr>
<th></th>
<th>NSW</th>
<th>QLD</th>
<th>SA</th>
<th>TAS</th>
<th>VIC</th>
</tr>
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<tr>
<td>NSW</td>
<td>1.0000000</td>
<td></td>
<td></td>
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<td>0.7958621</td>
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<td></td>
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<td>0.7117614</td>
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Kendall's $\tau$

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<th>SA</th>
<th>TAS</th>
<th>VIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>NSW</td>
<td>1.0000000</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>QLD</td>
<td>0.6527967</td>
<td>1.0000000</td>
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<td>SA</td>
<td>0.6160192</td>
<td>0.4364166</td>
<td>1.0000000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>TAS</td>
<td>0.3545376</td>
<td>0.2683777</td>
<td>0.4170962</td>
<td>1.0000000</td>
<td></td>
</tr>
<tr>
<td>VIC</td>
<td>0.7311607</td>
<td>0.5587940</td>
<td>0.7549099</td>
<td>0.4314016</td>
<td>1.0000000</td>
</tr>
</tbody>
</table>

- Strong dependence between the pairs (NSW, QLD), (NSW, VIC) and (VIC, SA) - markets that are well connected via interconnector transmission lines
- Significantly lower dependence between markets that are not directly interconnected, such as QLD and SA, or NSW
- Slightly higher correlation between TAS and VIC might be due to the fact that TAS is connected to VIC via an undersea inter-connector
Specify Marginal Distribution

Figure: Histogram for the pooled data vs. normal density (left panel) and Student-t density (right panel). Pooled data is taken for deseasonalized standardized log-prices for NSW, QLD, SA, TAS and VIC (daily data) from 01.01.2006 to 01.01.2010. The estimated number of degrees of freedom for the Student-t distribution for the pooled data corresponds to 3.
Specify Marginal Distribution

Symmetric generalized hyperbolic (SGH) family of distributions:

\[ f_X(x) = \frac{1}{\delta \sigma K_\lambda(\bar{\alpha})} \sqrt{\frac{\bar{\alpha}}{2\pi}} \left( 1 + \frac{x^2}{(\delta \sigma)^2} \right)^{\frac{1}{2}(\lambda - \frac{1}{2})} K_{\lambda - \frac{1}{2}} \left( \bar{\alpha} \sqrt{1 + \frac{x^2}{(\delta \sigma)^2}} \right) \]

- \( K_\lambda(\cdot) \) Bessel function
- \( \lambda \) and \( \bar{\alpha} \) are the shape parameters: \( \bar{\alpha} \neq 0 \) if \( \lambda \geq 0 \) and \( \delta \neq 0 \) if \( \lambda \leq 0 \)
  - Variance Gamma (VG) distribution: \( \bar{\alpha} = 0 \) and \( \lambda > 0 \)
  - Student-\( t \) distribution: \( \bar{\alpha} = 0 \) and \( \lambda < 0 \) (consider \( \lambda \leq -1 \) for \( \nu = -2\lambda \geq 2 \), std.dev. \( \sigma_X = \sigma \sqrt{\frac{\nu}{\nu - 2}} \))
  - Hyperbolic (HYP) distribution: \( \lambda = 1 \)
  - Normal Inverse Gaussian (NIG) distribution: \( \lambda = -0.5 \)
Specify Marginal Distribution

- All distributions from the SGH family provide a considerably better fit than the normal distribution.
- The Student-t assumption on the marginals provides the best overall fit to the data according to AD statistic.
- Fit GARCH(1,1) volatilities in

\[
X_t = a_0 + a_1 X_{t-1} + \varepsilon_t, \quad \text{where} \quad \varepsilon_t = \sigma_t u_t
\]

\[
\varepsilon_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \tag{2}
\]

for each of the marginals, assuming Student-t innovations.
Choice of copulas: static case

- Akaike information criterion (AIC):
  \[ AIC = -2L(\alpha; x_1, \ldots, x_T) + 2q \]
  
  favors:
  - Two-dim portfolios (10 possible combinations of NSW, QLD, SA, TAS and VIC markets):
    - Student-t copula is ranked first among all Archimedean and elliptical copulae in 9 out of 10 cases
    - Mixture Gumbel & survival Gumbel copula is ranked first overall in 8 out of 10 cases
    - Student-t copula outperforms at least one mixture model
  - Four-dim portfolios (NSW, QLD, SA, VIC):
    - Student-t copula outperforms at least one mixture model
    - Gumbel & survival Gumbel copula is superior to the remaining models
## Tail Dependence

<table>
<thead>
<tr>
<th>Copula</th>
<th>Student-t</th>
<th>Gumbel &amp; surv. Gumbel</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\nu$</td>
<td>$\rho$</td>
</tr>
<tr>
<td>(NSW, QLD)</td>
<td>2.60</td>
<td>0.84</td>
</tr>
<tr>
<td>(NSW, SA)</td>
<td>4.59</td>
<td>0.65</td>
</tr>
<tr>
<td>(NSW, TAS)</td>
<td>7.00</td>
<td>0.40</td>
</tr>
<tr>
<td>(NSW, VIC)</td>
<td>2.70</td>
<td>0.85</td>
</tr>
<tr>
<td>(QLD, SA)</td>
<td>6.01</td>
<td>0.45</td>
</tr>
<tr>
<td>(QLD, TAS)</td>
<td>14.15</td>
<td>0.28</td>
</tr>
<tr>
<td>(QLD, VIC)</td>
<td>3.69</td>
<td>0.65</td>
</tr>
<tr>
<td>(SA, TAS)</td>
<td>6.92</td>
<td>0.46</td>
</tr>
<tr>
<td>(SA, VIC)</td>
<td>2.63</td>
<td>0.86</td>
</tr>
<tr>
<td>(TAS, VIC)</td>
<td>4.56</td>
<td>0.56</td>
</tr>
<tr>
<td>(NSW, QLD, SA, VIC)</td>
<td>3.62</td>
<td>0.76</td>
</tr>
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</table>
Time-varying estimation

- Static case: estimate the dependence parameter at once based on the whole series of observations.

- Time-varying case:
  - Estimate the dependence parameter by using subsets of size $n$ of deseasonalized log-prices, that is a moving window of size $n$, $\{\hat{X}_t\}_{t=s-n+1}^s$ scrolling in time for $s = n, \ldots, T$.
  - It generates a time-series for the dependence parameter $\{\hat{\theta}_t\}_{t=n}^T$ and time-series of VaR: $\{\hat{VaR}_t\}_{t=n}^T$. 

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Student-t copula dependence parameter: Time-varying

Dependence parameter for (NSW, QLD) estimated using Student-t copula with Student-t marginals

Dependence parameter for (NSW, QLD, SA, VIC) estimated using Student-t copula with Student-t marginals

Figure: Upper panel: Copula dependence parameter $\hat{\theta}$ estimated using a bivariate Student-t copula with Student-t marginals for (NSW, QLD). Lower panel: Copula dependence parameter $\hat{\theta}$ estimated using a four-dimensional Student-t copula with Student-t marginals for (NSW, QLD, SA, VIC)
Mixture Gumbel & survival Gumbel: Time-varying

Theta 1 dependence parameter for (NSW, QLD, SA, VIC) from the mixture Gumbel & survival Gumbel copula with Student-t marginals

Theta 2 dependence parameter for (NSW, QLD, SA, VIC) from the mixture Gumbel & survival Gumbel copula with Student-t marginals

Theta 3 mixture parameter for (NSW, QLD, SA, VIC) from the mixture Gumbel & survival Gumbel copula with Student-t marginals

Figure: Copula dependence parameters $\hat{\theta}_1$ (upper left panel), $\hat{\theta}_2$ (upper right panel) and mixture parameter $\hat{\theta}_3$ (low panel) estimated using a mixture model Gumbel & survival Gumbel copula with Student-t marginals for (NSW, QLD, SA, VIC).
Consider the value of portfolio

\[ V_t = \sum_{j=1}^{d} w_j \exp(X_{j,t}) \]

of investment \( w = (w_1, \ldots, w_d)^\top \) in the spot contracts of (NSW, QLD, SA, VIC)

The VaR of a portfolio at time \( t \) and level \( \alpha \) is defined as the \( \alpha \)-quantile from the distribution of the portfolio value:

\[ \text{VaR}_t(\alpha) = F_{V_t}^{-1}(\alpha). \]  

VaR is computed at level \( \alpha \in \{0.9, 0.95, 0.99, 0.995, 0.999\} \) (i.e. probability of exceedance \( p = 1 - \alpha \))
Backtesting

- Compare the estimated values for the VaR with the true realizations $V_t$ of the portfolio value

  - *Binary loss function* (Lopez(1998)):
    
    $$L_t = \begin{cases} 
    1, & V_t > \text{VaR}_t \\
    0, & V_t \leq \text{VaR}_t 
    \end{cases}.$$
    
    (4)

  - *Quadratic probability score* (QPS):
    
    $$QPS = \frac{2}{n} \sum_{t=1}^{n} (L_t - p)^2.$$
    
    (5)

  - *Size-adjusted loss function* (Blanco(1999)):
    
    $$L_t = \begin{cases} 
    (V_t - \text{VaR}_t)/\text{VaR}_t, & V_t > \text{VaR}_t \\
    0, & V_t \leq \text{VaR}_t 
    \end{cases}.$$
    
    (6)
Backtesting: 2-dim (NSW, QLD)

Figure: Actual portfolio value and estimated VaR in log-scale for portfolio $w = (1, 1)^T$ of spot contracts in NSW and QLD. VaR is estimated at conf. levels $\alpha \in \{0.9, 0.95, 0.99, 0.995, 0.999\}$ using Gaussian copula model with normal marginals (upper panel) and mixture Gumbel & survival Gumbel with Student-t marginals (lower panel). The exceedances are computed at level $\alpha = 0.995$. 

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### Backtesting: 2-dim case (NSW, QLD)

<table>
<thead>
<tr>
<th>Copula</th>
<th>Measure</th>
<th>90%</th>
<th>95%</th>
<th>99%</th>
<th>99.5%</th>
<th>99.9%</th>
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<tbody>
<tr>
<td>Gaussian</td>
<td>Exceptions</td>
<td>0.1203</td>
<td>0.0766</td>
<td>0.0365</td>
<td>0.0301</td>
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<tr>
<td></td>
<td>QPS</td>
<td><strong>0.2125</strong></td>
<td><strong>0.1428</strong></td>
<td>0.0717</td>
<td>0.0596</td>
<td>0.0455</td>
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<tr>
<td></td>
<td>Blanco</td>
<td>128.39</td>
<td>124.95</td>
<td>106.26</td>
<td>101.52</td>
<td>87.28</td>
</tr>
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</table>

<table>
<thead>
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<th>Copula</th>
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<th>95%</th>
<th>99%</th>
<th>99.5%</th>
<th>99.9%</th>
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<tbody>
<tr>
<td>Student-t</td>
<td>Exceptions</td>
<td>0.1413</td>
<td>0.0802</td>
<td>0.0356</td>
<td>0.0246</td>
<td>0.0119</td>
</tr>
<tr>
<td></td>
<td>QPS</td>
<td>0.2461</td>
<td>0.1494</td>
<td><strong>0.0699</strong></td>
<td><strong>0.0488</strong></td>
<td>0.0237</td>
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<td>Blanco</td>
<td><strong>123.86</strong></td>
<td><strong>120.73</strong></td>
<td><strong>105.65</strong></td>
<td><strong>98.83</strong></td>
<td>77.40</td>
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<table>
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<th>Copula</th>
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<th>99%</th>
<th>99.5%</th>
<th>99.9%</th>
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<td>G.&amp; surv.G.</td>
<td>Exceptions</td>
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<td>128.80</td>
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<td>106.46</td>
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<td><strong>71.09</strong></td>
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**Table:** Results for fraction of exceptions, quadratic probability score (QPS) and the size-adjusted loss function Blanco (1999) for a stylized portfolio of holding \( w = (1, 1) \)\(^\top\) spot contracts in both NSW and QLD market. Results are reported for the considered VaR levels 90%, 95%, 99%, 99.5% and 99.9%. The results for the best model with respect to QPS score and Blanco (1999) loss function are indicated in bold letters.
### Backtesting: 2-dim case (NSW, QLD)

<table>
<thead>
<tr>
<th>Copula</th>
<th>Test</th>
<th>90%</th>
<th>95%</th>
<th>99%</th>
<th>99.5%</th>
<th>99.9%</th>
</tr>
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<tbody>
<tr>
<td>Gaussian</td>
<td>Kupiec</td>
<td>0.012**</td>
<td>0.000***</td>
<td>0.000***</td>
<td>0.000***</td>
<td>0.000***</td>
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<tr>
<td></td>
<td>( LR_{unc} )</td>
<td>4.76**</td>
<td>14.13***</td>
<td>46.22***</td>
<td>64.11***</td>
<td>109.03***</td>
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<tr>
<td>Student-t</td>
<td>Kupiec</td>
<td>0.000***</td>
<td>0.000***</td>
<td>0.000***</td>
<td>0.000***</td>
<td>0.000***</td>
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<tr>
<td></td>
<td>( LR_{unc} )</td>
<td>18.67***</td>
<td>17.97***</td>
<td>43.60***</td>
<td>43.46***</td>
<td>40.61***</td>
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<tr>
<td>G.&amp; surv.G.</td>
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<td>0.000***</td>
<td>0.000***</td>
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<tr>
<td></td>
<td>( LR_{unc} )</td>
<td>17.89***</td>
<td>17.97***</td>
<td>43.60***</td>
<td>46.73***</td>
<td>35.72***</td>
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**Table:** Results for Kupiec test (p-value) and likelihood ratio test for unconditional coverage \( LR_{unc} \) for a stylized portfolio of \( w = (1, 1)^\top \) spot contracts in the NSW and QLD market. The test is conducted for the considered VaR levels 90%, 95%, 99%, 99.5% and 99.9%. The asterisk denote rejection of an appropriate model specification at the * - 10%, ** - 5% and *** - 1% significance level. The results for the best model with respect to \( LR_{unc} \) are indicated in bold letters.
Backtesting: 2-dim case (NSW,QLD)

<table>
<thead>
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<th>Copula</th>
<th>Test</th>
<th>90%</th>
<th>95%</th>
<th>99%</th>
<th>99.5%</th>
<th>99.9%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gaussian</td>
<td>Runs test</td>
<td>-12.18***</td>
<td>-8.59***</td>
<td>-0.90</td>
<td>-0.53</td>
<td>0.10</td>
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<tr>
<td></td>
<td>$LR_{ind}$</td>
<td>103.08***</td>
<td>44.69***</td>
<td>0.19</td>
<td>0.00</td>
<td>1.17</td>
</tr>
<tr>
<td>Student-t</td>
<td>Runs test</td>
<td>-1.40</td>
<td>-1.00</td>
<td>-0.10</td>
<td>0.21</td>
<td>-0.91</td>
</tr>
<tr>
<td></td>
<td>$LR_{ind}$</td>
<td>1.52</td>
<td><strong>0.58</strong></td>
<td><strong>0.13</strong></td>
<td><strong>1.36</strong></td>
<td>0.31</td>
</tr>
<tr>
<td>G. &amp; surv.G.</td>
<td>Runs test</td>
<td>-0.97</td>
<td>-1.00</td>
<td>-0.10</td>
<td>0.26</td>
<td>-1.05</td>
</tr>
<tr>
<td></td>
<td>$LR_{ind}$</td>
<td><strong>0.68</strong></td>
<td><strong>0.58</strong></td>
<td><strong>0.13</strong></td>
<td>1.47</td>
<td><strong>0.27</strong></td>
</tr>
</tbody>
</table>

**Table:** Results for runs test and likelihood ratio test ($LR_{ind}$) for independence of VaR exceptions for stylized portfolio of holdings $w = (1, 1)^\top$ spot contracts in NSW and QLD market. The tests are conducted for the considered confidence levels 90%, 95%, 99%, 99.5% and 99.9%. The asterisk denote rejection of an appropriate model specification at the * -10%, ** - 5% and *** - 1% significance level. The results for the best model with respect to $LR_{ind}$ are indicated in bold letters.
Backtesting: 2-dim case (NSW, QLD)

- All of the considered models tend to generally underestimate the Value-at-Risk.
- Mixture Gumbel & survival Gumbel model leads to the number of exceptions which is closer to the true number of exceptions, followed by the Student-t and the Gaussian copula.
- Exceptions: At the 90% and 95% confidence levels the Gaussian copula model provides the best results; for higher VaR levels such as 99%, 99.5% and 99.9%, the Student-t and the mixture copula models provide the lowest number of VaR exceedances and a better QPS score.
- Blanco: Student-t copula provides better results at 90%, 95%, 99%, 99.5% levels, while the mixture Gumbel & survival Gumbel copula models provide better results at 99.9%.
- Clustering of VaR exceptions: Student-t and Mixture copulas clearly outperform the Gaussian copula.
Backtesting: 4-dim case (NSW, QLD, SA, VIC)

- All of the considered models tend to generally underestimate the Value-at-Risk: The number of VaR violations is higher than could be expected under an appropriate risk quantification.

- Mixture Gumbel & survival Gumbel model provides results closer to the true number of violations, followed by the Student-t and the Gaussian copula.

- QPS and Blanco: At the 90%, 95%, 99% and 99.5% confidence levels the mixture Gumbel & survival Gumbel copula model provides the best results; at 99.9% level, the Student-t provides a lowest number of VaR exceedances and a better QPS score.

- Clustering of VaR exceptions: Student-t and Mixture copulas clearly outperform the Gaussian copula.
Contributions and Findings

- Study the dependence structure between spot electricity prices across Australian regional electricity markets in NSW, QLD, SA, TAS and VIC.

- Combine a GARCH model to capture the time-varying volatilities in the regional markets with various copulae to capture the dependence structure between the markets.

- We find positive dependence between prices from all of the considered markets, the strongest dependence is exhibited between markets that are well connected via interconnector transmission lines such as NSW and QLD; NSW and VIC; SA and VIC.

- Despite the introduction of additional interconnectors during the considered time period, the dependence between considered regional markets has not increased significantly.

- Student-t copula outperforms all other Archimedean and elliptical copulae indicating some degree of symmetric tail dependence.
Contributions and Findings

- Overall best results are obtained using mixture models due to their ability of also capturing asymmetric dependence in the tails of the distribution: Gumbel & survival Gumbel copula.

- The performance of copula models is tested in a risk management application where we estimate the Value-at-Risk for a bivariate and multivariate portfolio of holding electricity spot contracts in different markets: Due to the spiky and extreme volatile behavior of electricity spot prices none of the considered models could provide an appropriate specification of the risk.

- Overall, the mixture Gumbel & survival Gumbel copula model in combination with Student-t marginals performs best, while the Student-t copula model (also with Student-t marginals) yields results that are only slightly worse.

- Both models outperform the Gaussian copula model, in particular with respect to the independence of VaR violations and the estimated VaR in the more extreme tails.
Thank you very much for your attention!